

CAT

Common Admission Test

A comprehensive book for CAT aspirants

Jaya Ghosh

B.Sc. (Maths), MCA, MBA (HR)

- ▶ Quantitative Aptitude
- ▶ Data Interpretation
- ▶ Logical Reasoning
- ▶ Verbal Ability and Reading Comprehension

The
Gen X
Series



**CAT 2014
Fully Solved
1 Mock Test
Paper**

2015

CAT

A comprehensive book for CAT aspirants and useful for various other competitive entrance exams like MCA, NIFT, and Hotel Management

- Tips and formulae for quick learning and revision
- Concepts explained through solved examples in simple and lucid language
- Multiple Choice Questions provided with Answer Key and Explanatory Notes
- Three Practice Assessment Exercises provided after every section for self evaluation
- CAT 2014 Solved Paper included for understanding the pattern and one Mock Test Paper for practice

Jaya Ghosh
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PUBLISHER'S NOTE

V&S Publishers, after the grand success of a number of Academic and General books, is pleased to bring out a series of books, *The Gen X series* which means **generating Xcellence in generation X** and which focuses on the smart learning and skills development of students. Under the series we are going to publish a precise and functional book, *CAT 2015 – A comprehensive book for CAT aspirants*, which has been exclusively designed to sort out problems of the candidates who aspire to get admission to the top management schools across the country and build a strong career in the field of management.

To go through a prominent entrance exam like Common Admission Test (CAT), the aspirants must study all the topics covered in the syllabus and make a tireless effort during preparation. This exam not only tests their subjective knowledge but also checks the pace of answering the questions i.e. how fast a candidate is able to grasp and solve the given problem. This kind of test requires careful attention towards comprehension of concepts, thorough practice, and application of the concepts in a very fast manner.

The book has been divided into four sections namely **Quantitative Aptitude, Data Interpretation, Logical Reasoning, Verbal Ability and Reading Comprehension**. In each section all the concepts have been explained through solved examples and important **formulae and tips** have been given to understand the problems in no time. To enhance the problem solving skills of candidates, **Multiple Choice Questions (MCQs) with detailed solutions** are given in the end of every chapter. After every section, **Three Practice Assessments** have been provided to evaluate the level of self preparation. **CAT 2014 Solved Paper to comprehend the pattern and format of exam, and one Mock Test Paper for Practice have been given in the last of the book.**

So, a candidate should make an honest effort to qualify the entrance because sincerity is always rewarded. We wish you success in the examination and a very bright future in the field of management.

Best of Luck!!!

ACKNOWLEDGEMENT

This book would not have been possible without the support and encouragement of my husband, Mr. Purnendu Ghosh.

I am deeply indebted to V & S Publishers for giving me the opportunity to write this book. I am thankful to the editorial and production staff of V & S Publishers for doing such a good job. I avail this opportunity to convey my sincere thanks to Mr. Sahil Gupta, Director of V & S Publishers, for the efforts put in by him to keep the book error-free.

Any suggestion/feedback to enhance the quality of the book will be gracefully acknowledged.

— Jaya Ghosh

INTRODUCTION

Why MBA?

An MBA degree these days is a pre-requisite to a corporate career. Unless you're not an MBA, very few companies would be willing to give you a managerial position. MBA is the most sought after not just because it is lucrative position but because of a host of reasons. The kind of expert training and exposure you need to survive and succeed in the corporate sector is provided by MBA.

Not just that, the network of high achievers and stellar minds that you create there go in a long way in life.

In this competitive world, you should not leave anything on fate or chance. So, whether you want to become an entrepreneur or want a dream job, MBA is the way to go!

The experience gained at top business schools shall remain with you for the rest of your life subconsciously influencing your thought and showing you the right path.

How to prepare for CAT?

If someone says that an average candidate can prepare for CAT in just 80 hours, how many of you will accept it? You must have a strong feeling in your mind that CAT is one of the toughest tests in the country.

Till 2008, when CAT was paper based, 35% of the maximum marks used to assure you a very good percentile. Now if we talk about the present scenario, we can assume how tough the competition is?

The candidates are so much worried and in such a pressure that they can't even answer 35% of the questions which is not even pass mark in lower grades.

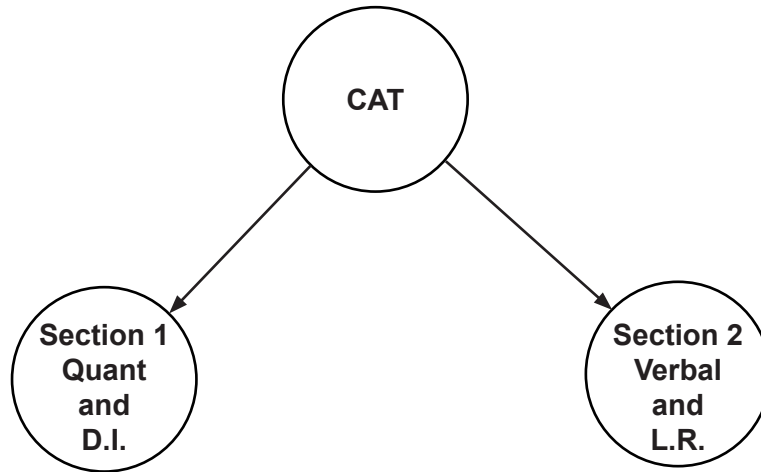
Having belief in your preparation and with fresh mindset, you can easily answer more than 50% of CAT questions without having any prior knowledge. Then why CAT is difficult?

Understand that CAT is not mathematics or English language. It includes simple basics with interpretation, understanding, visualization, observation and approach.

The first section of CAT, which contains quantitative ability and Data Interpretation, can easily be prepared in 40 – 50 hours.

Preparing the whole quantitative ability section is as simple as knowing 50 – 60 basic points on mathematics. Complex mathematical questions will not appear in CAT so it does not matter whether you are a student of mathematics background or not. How much does it take to learn these 50 – 60 points? At the max 4 – 5 hours.

Keep the following points in mind while taking the test. Not all the questions that appear in CAT require prior knowledge.



Please note that 30 – 40% of CAT questions can be answered just by understanding, not by using any formula. Even in case if there is some question that belongs to a particular topic, be it algebra, number system or geometry, then even that can be solved with very basic knowledge and understanding of maths.

Catch The Nerve Of The Paper Setter!!!

When you read a question, try to understand why that question is being asked? Always remember one thing, the paper setter does not want to test your memory or they are not interested in knowing how good you are in Maths and English. Try to understand one thing that top B-schools are not looking for scholars in Maths or English veterans. You just need to have good understanding, analyzing and observing skills along with right approach to impress them. So it would be justified to say that to crack CAT with more than 95% percentile what you all need to do is to learn the basics of mathematics and to lay more emphasis on the approach we choose to solve various questions.

SYLLABUS AND SCHEME OF TEST

It is very important for an MBA aspirants to know the detailed syllabus for good preparation. Common Admission Test (CAT) syllabus includes Quantitative Aptitude, Verbal Ability, Reasoning Ability which includes Logical Reasoning and Critical Reasoning.

The aspirants who prepare for the CAT exam to get admission to IIMs and other institutes of repute have many doubts about the format of the exam.

DETAILED PATTERN OF VARIOUS MBA ENTRANCE EXAMS

Exams	Sections	Total Questions	Total Time (duration)	Paper/Computer Based
CAT	QA+DI VA+AR	60	2 Hrs. 20 min.	Computer based
XAT	QA+DI AR+DM VA+LR Essay Writing	100 +	2 Hours	Paper based
SNAP	QA+DI AR+LR VA+GK	135	2 Hours	Paper based
IIFT	QA+DI VA+LR GK	100 +	2 Hours	Paper based
MAT	QA+DI LR+VA GK	200	2 Hours 30 min.	Paper/Computer based

Abbreviations:

QA* – Quantitative Aptitude

DI* – Data Interpretation

GK* – General Knowledge

VA* – Verbal Ability

AR* – Analytical Reasoning

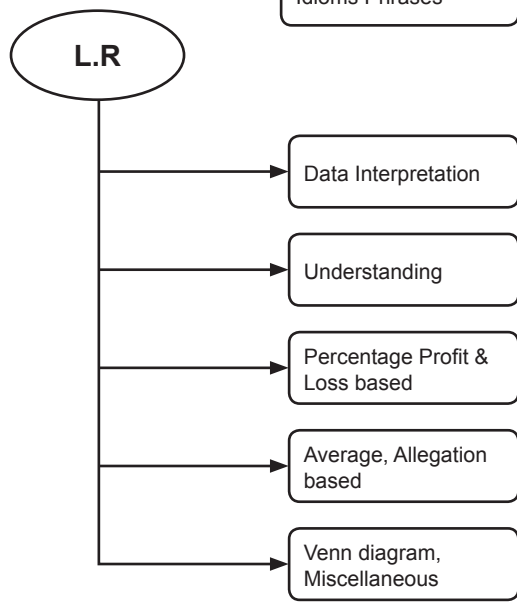
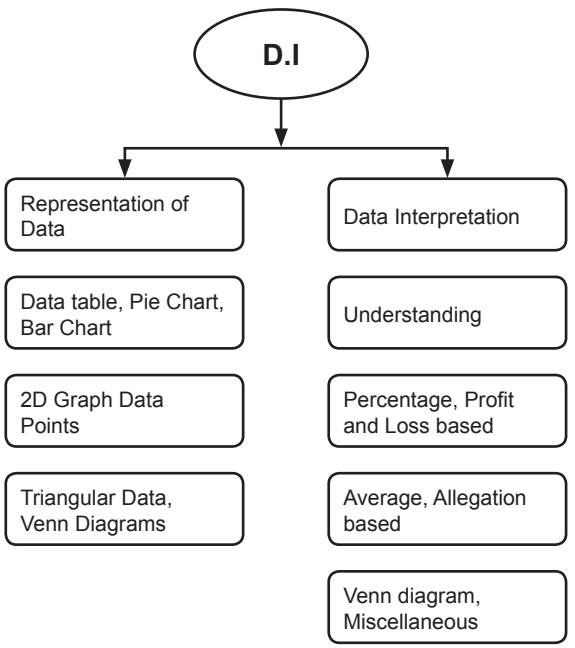
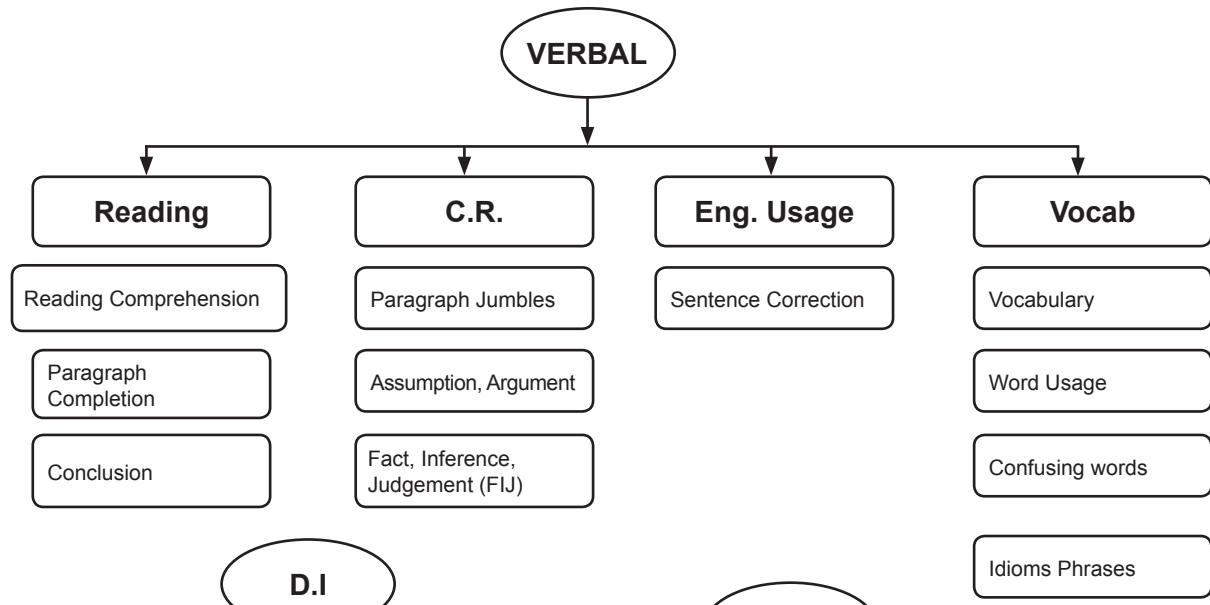
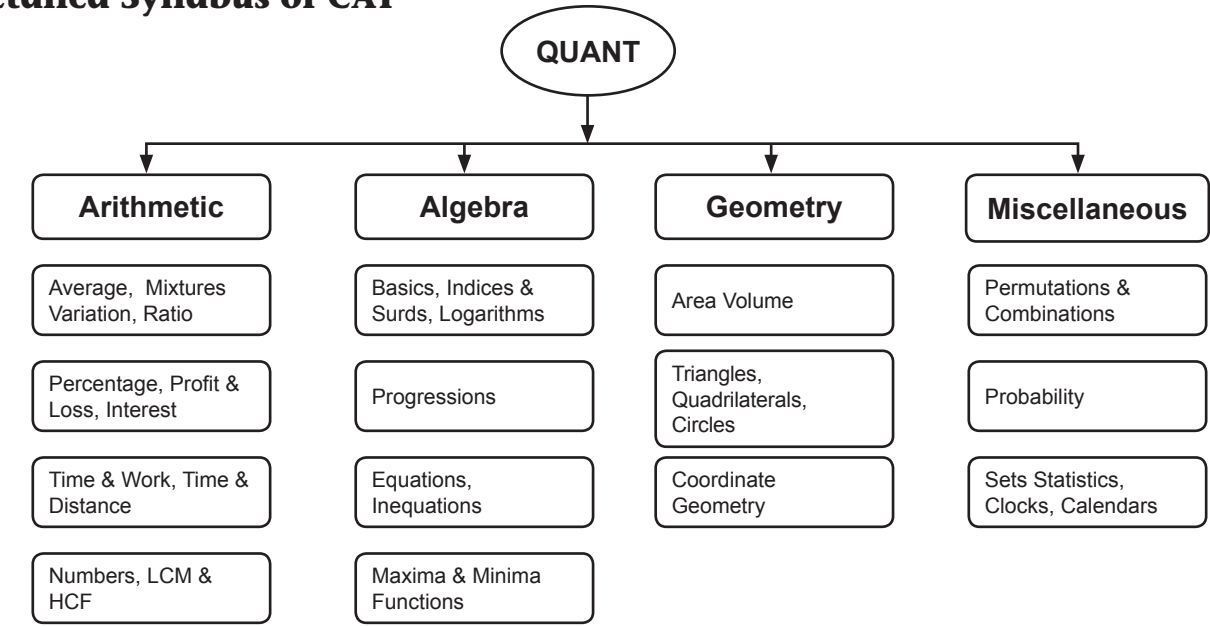
DM* – Decision Making

LR* – Logical Reasoning

ELIGIBILITY CRITERIA

EXAM	ELIGIBILITY
CAT	At least 50% marks at the graduation level (for SC or ST candidates 45% marks required). Final year graduates can also apply. No Age limit.
MAT	Graduate from a recognized university. Final year candidates can also apply. No age limit. No minimum marks required in graduation.
SNAP	Graduate from a recognized university with a minimum score of 50%. For candidates belonging to SC/ST category a score of 45% in graduation is essential to apply for the SNAP test.
XAT	Minimum bachelor's degree under the 10 + 2 + 3 pattern of education. Final year candidates can also apply.
NMAT	Graduate from a recognized university with a minimum score of 50%. Final year candidates are also eligible.
IIFT	Recognized Bachelor's degree of minimum 3 years' duration in any discipline. Final year candidates can also apply. No age limit.

Detailed Syllabus of CAT



NEW PATTERN CAT 2014

In the new CAT 2014 pattern, you could move between sections.

This is a very good change. In the conventional pen and paper based exam, a candidate had to go through questions and attempt the easiest first. In CAT, the biggest disadvantage was that the sections were allowed to be attempted one by one. Now, you can skip the sections.

Every student has some strengths and some weakness. Whether you are very good in quantitative questions (Math based) or Verbal and comprehension questions (language based), make sure you attempt the section in which you are the most proficient. That way, you will grab more marks in less time in the first half of the test. Then move to the section you are weaker in. Go steady in that section, understand and solve the rest of questions.

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SECTION 1

QUANTITATIVE
APTITUDE

NUMBER SYSTEM

Learning Objectives

After going through this chapter, you will be able to learn:

- What is the number system
- Practical applications of the number system
- Tips to score better in the number system

Introduction

Number System

Number system is the key concept in every branch of mathematics. The use and scope of number system is unlimited. It is the backbone of any Competitive exam. The correct understanding will help you to solve different and complex problems that appear in these examinations.

We are giving here under the 'number chart' which is self explanatory in nature and use for practical application.

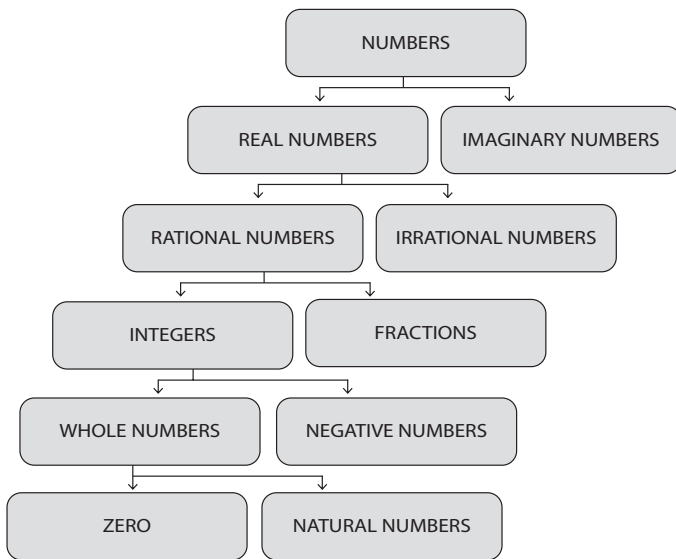


Fig 1.1 Number Chart

Types of Questions

Questions asked from this topic are of the following two kinds:

- I. Based on definitions or properties of numbers
- II. Based on concepts
 - ✓ **Based on definitions:** Questions of this kind are based on definitions and properties of different kinds of numbers, involving formulae and some very basic calculations.
 - ✓ **Based on concepts:** The other kind of questions are based on the following concepts:
 - LCM and HCF
 - Divisibility

- Divisibility rules (for base other than 10)
- Number of divisors
- Number of exponents
- Remainders
- Base system
- Unit digits
- Tens digit

Complex Numbers

A complex number is a number that can be put in the form $a + bi$, where 'a' and 'b' are real numbers and i is called the imaginary unit, where $i^2 = -1$.

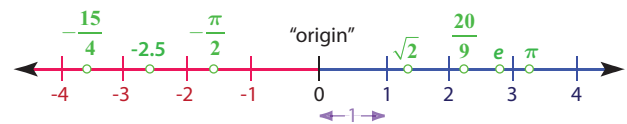
Real Numbers

In mathematics, a real number is a value that represents a quantity along a continuous line. The real numbers include all the rational numbers, such as the integer -5 and the fraction $\frac{4}{3}$, and all the irrational numbers such as $\sqrt{2}$ (1.41421356... the square root of two, an irrational algebraic number) and π (3.14159265..., a transcendental number). Real numbers can be thought of as points on an infinitely long line called the number line or real line, where the points corresponding to integers are equally spaced.

The Real Number Line

The Real Number Line is like an actual geometric line.

A point is chosen on the line to be the 'origin', points to the right will be positive, and points to the left will be negative.

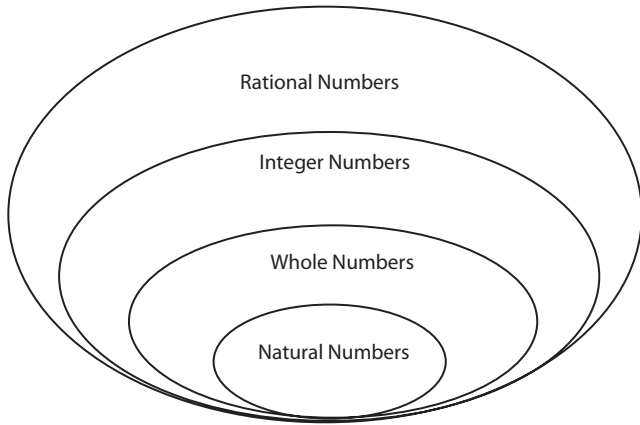


Now, real number line can be divided into two categories: rational numbers and irrational numbers.

Rational Numbers (Q)

A Rational Number of the form $\frac{p}{q}$ or a number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a Rational Number.

$\frac{1}{2}$, $\frac{-2}{5}$ are examples of rational numbers



Irrational Numbers

A number which cannot be put in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called an irrational number.

A number whose decimal expression is non-terminating and non-recurring is called an irrational number.

$\sqrt{2}$, $\sqrt{-3}$, $\sqrt{-5}$, π (Pi) are irrational numbers.

Rational numbers can further be subdivided into two parts—integers and fractions.

Integers

There are different types of numbers, natural numbers, whole numbers, fractions, decimals etc.

Natural Numbers = 1, 2, 3, 4, 5...

Whole Numbers = 0, 1, 2, 3, 4...

Fractions = $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$... These numbers lie between the two whole numbers.

Decimals = 0.1, 0.2, 0.3 ... These numbers also lie between the two whole numbers.

Whole numbers along with the negative numbers are called **Integers**.

The numbers ..., -4, -3, -2, -1, 0, 1, 2, 3, 4 ... etc. are called integer.

1, 2, 3, 4... are **Positive integer**.

-1, -2, -3... are **Negative numbers**.

Fractions

A fraction includes two parts, numerator and denominator $\frac{3}{7}$, $\frac{9}{5}$, $\frac{11}{7}$ etc.

Fractions are primarily of five types:

- Proper Fraction
- Improper Fraction
- Mixed Fraction
- Compound Fraction
- Complex Fraction

Concept of Unit Digits

For the concept of identifying the unit digit, we have to first familiarize with the concept of cyclicity. Cyclicity of any

number is about the last digit and how they appear in a certain defined manner. Let's take an example to clear this thing:

The cyclicity chart of 2 is:

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

Have a close look at the above. You would see that as 2 is multiplied every time with its own self, the last digit changes. On the 4th multiplication, 2^5 have the same unit digit as 2^1 . This shows us the cyclicity of 2 is 4, i.e. after every fourth multiplication, the unit digit will be two.

Cyclicity table:

The cyclicity table for numbers is given as below:

Number	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

How did we figure out the above?

Now let us see and the concept of cyclicity to calculate the Unit digit of a number.

What is the unit digit of the expression 4^{45} ? Now we have two methods to solve this but we choose the best way to solve it i.e. through cyclicity

We know the cyclicity of 4 is 2

Have a look:

$$4^1 = 4$$

$$4^2 = 16$$

$$4^3 = 64$$

Here the digit 4 comes again to the end when 4 raised to the power of 3, so it is clear that the cyclicity of 4 is 2. Now with the cyclicity number i.e. with 2 divide the given power i.e. $45/2$. In this case remainder the remainder will be 1.

i.e. $4^1 = 4$

So, the unit digit in this case is 4.

Solved Examples:

Example 1:

The digit in the unit place of the number $7^{95} \times 3^{58}$ is:

(a) 7

(b) 2

(c) 6

(d) 4

Example 6:

It is being given that $(2^{32} + 1)$ is completely divisible by a whole number. Which of the following numbers is completely divisible by this number?

- (a) $(2^{16} + 1)$ (b) $(2^{16} - 1)$
 (c) (7×2^{23}) (d) $(2^{96} + 1)$

Solution: Option (d) is correct.

Explanation:

Let $2^{32} = x$. Then, $(2^{32} + 1) = (x + 1)$.

Let $(x + 1)$ be completely divisible by the natural number N . Then, $(2^{96} + 1) = [(2^{32})^3 + 1] = (x^3 + 1) = (x + 1)(x^2 - x + 1)$, which is completely divisible by N , since $(x + 1)$ is divisible by N .

Example 7:

Evaluate $\frac{854 \times 854 \times 854 - 276 \times 276 \times 276}{854 \times 854 + 854 \times 276 + 276 \times 276} = ?$

- (a) 1130 (b) 578
 (c) 565 (d) 1156

Solution: Option (b) is correct.

Explanation:

Given Exp. = $\frac{(a^3 - b^3)}{(a^2 + ab + b^2)} = (a - b) = (854 - 276) = 578$

Number Series**Arithmetic Progression (A.P.)**

A sequence is said to be in Arithmetic Progression when they increase or decrease by a constant number. This constant number is called the common difference (c.d.) of the arithmetic progression.

- N^{th} term of an A.P. (t_n) = $a + (n - 1)d$
- Sum of the first n term of an A.P. (S_n) = $\{2a + (n - 1)d\}$

Examples:

- 1, 3, 5, 7... c.d. = 2
 -7, -3, 1, 5, 9 ... c.d. = 4

Geometric Progression

A sequence is said to be in Geometric Progression, if the ratio between any two adjacent numbers in the sequence is constant (non zero). This constant is said to be common ratio (c.r.)

Examples:

- 1, 2, 4, 8c.r. = 2
- 1, 1/2, 1/4, 1/8.....c.r. = 1/2
- The N^{th} term of G.P. (t_n) = ar^{n-1}

Example 8:

The sum of first 45 natural numbers is:

- (a) 1035 (b) 1280
 (c) 2070 (d) 2140

Solution: Option (a) is correct.

Explanation:

Let $S_n = (1 + 2 + 3 + \dots + 45)$

This is an A.P. in which $a = 1$, $d = 1$, $n = 45$ and $l = 45$

$$\therefore S_n = n/2(a + l) = 45/2 \times (1 + 45) = (45 \times 23) = 1035$$

Required sum = 1035.

Factorization

It is the process of splitting any number into the form of its basic prime factors.

For example: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

24 is expressed in the factorised form in terms of its basic prime factors. This is the factorisation form of 24.

Example 9:

If n is a positive integer and $(n + 1)(n + 3)$ is odd, then $(n + 2)(n + 4)$ must be a multiple of which one of the following?

- (a) 3 (b) 5
 (c) 6 (d) 8

Solution: Option (d) is correct.

$(n + 1)(n + 3)$ is odd only when both $(n + 1)$ and $(n + 3)$ are odd. This is possible only when n is even.

Hence, $n = 2m$, where m is a positive integer. Then,

$$(n + 2)(n + 4) = (2m + 2)(2m + 4) = 2(m + 1)2(m + 2) = 4(m + 1)(m + 2)$$

= $4x$ (product of two consecutive positive integers, one which must be even)

= $4x$ (an even number), and this equals a number that is at least a multiple of 8.

Example 10:

A girl wrote all the numbers from 100 to 200. Then she started counting the number of one's that has been used while writing all these numbers. What is the number that she got?

- (a) 111 (b) 119
 (c) 120 (d) 121

Solution: Option (c) is correct.

From 100 to 200 there are 101 numbers. There are 100 1's in the hundred places. 10 1's in tens place 10 1's in unit place. Thus the answer is $100 + 10 + 10 = 120$.

Important Formulae

- Sum of all the first n natural numbers = $\frac{n(n+1)}{2}$
- Sum of first n odd numbers = n^2
- Sum of first n even numbers = $n(n+1)$
- Sum of squares of first n natural numbers = $\frac{n(n+1)(n+2)}{6}$
- Sum of cubes of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$
- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)$
- $(a^2 + b^2 + c^2 - ab - bc - ac)$, when $a + b + c = 0$,
- then $a^3 + b^3 + c^3 = 3abc$.

Practice Exercise

1. What is the unit digit in the product $(3^{65} \times 6^{59} \times 7^{71})$?
 (a) 1 (b) 2
 (c) 4 (d) 6
2. On dividing a number by 68, we get 269 as quotient and 0 as remainder. On dividing the same number by 67, what will the remainder?
 (a) 0 (b) 1
 (c) 2 (d) 3
3. On multiplying a number by 7, the product is a number each of whose digits is 3. The such smallest number is:
 (a) 47619 (b) 47719
 (c) 48619 (d) 47649
4. If x and y are the two digits of the number 653 xy such that this number is divisible by 80, then $x + y = ?$
 (a) 2 or 6 (b) 4
 (c) 4 or 8 (d) 8
5. How many terms are there in the G.P. 3, 6, 12, 24, ... 384?
 (a) 8 (b) 9
 (c) 10 (d) 11
6. If x and y are positive integers such that $(3x + 7y)$ is a multiple of 11, then which of the following will be divisible by 11?
 (a) $4x + 6y$ (b) $x + y + 4$
 (c) $9x + 4y$ (d) $4x - 9y$
7. In a division sum, the remainder is 0. A student mistook the divisor by 12 instead of 21 and obtained 35 as quotient. What is the correct quotient?
 (a) 0 (b) 12
 (c) 13 (d) 20
8. Find the value of:
 $2 + 2^2 + 2^3 + \dots + 2^9 = ?$
 (a) 2044 (b) 1022
 (c) 1056 (d) None of these
9. The sum of how many terms of the series $6 + 12 + 18 + 24 + \dots$ is 1800?
 (a) 16 (b) 24
 (c) 20 (d) 18
10. What is the unit digit in $(7^{95} - 3^{58})$?
 (a) 0 (b) 4
 (c) 6 (d) 7
11. $(x^n - a^n)$ is completely divisible by $(x - a)$, when
 (a) n is any natural number
 (b) n is an even natural number
 (c) n is an odd natural number
 (d) n is prime
12. Which of the following numbers will completely divide $(3^{25} + 3^{26} + 3^{27} + 3^{28})$?
 (a) 11 (b) 16
 (c) 25 (d) 30
13. n is a whole number which when divided by 4 gives 3 as remainder. What will be the remainder when $2n$ is divided by 4?
 (a) 3 (b) 2
 (c) 1 (d) 0
14. $476**0$ is divisible by both 3 and 11. The non-zero digits in the hundred's and ten's places are respectively:
 (a) 7 and 4 (b) 7 and 5
 (c) 8 and 5 (d) None of these
15. Evaluate:
 $9 + \frac{3}{4} + 7 + \frac{2}{17} - (9 + \frac{1}{15}) = ?$
 (a) $7 + \frac{719}{1020}$ (b) $9 + \frac{817}{1020}$
 (c) $9 + \frac{719}{1020}$ (d) $7 + \frac{817}{1020}$
16. On dividing 2272 as well as 875 by 3-digit number N , we get the same remainder. The sum of the digits of N is:
 (a) 10 (b) 11
 (c) 12 (d) 13
17. A boy multiplied 987 by a certain number and obtained 559981 as his answer. If in the answer both 98 are wrong and the other digits are correct, then the correct answer would be:
 (a) 553681 (b) 555181
 (c) 555681 (d) 556581
18. Which one of the following is the common factor of $(47^{43} + 43^{43})$ and $(47^{47} + 43^{47})$?
 (a) $(47 - 43)$ (b) $(47 + 43)$
 (c) $(47^{43} + 43^{43})$ (d) None of these
19. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, what is the dividend?
 (a) 4236 (b) 4306
 (c) 4336 (d) 5336
20. Evaluate: $\{(476 + 424)^2 - 4 \times 476 \times 424\} = ?$
 (a) 2906 (b) 3116
 (c) 2704 (d) 2904
21. Which of the following numbers will completely divide $(4^{61} + 4^{62} + 4^{63} + 4^{64})$?
 (a) 3 (b) 10
 (c) 11 (d) 13
22. $(1^2 + 2^2 + 3^2 + \dots + 10^2) = ?$
 (a) 330 (b) 345
 (c) 365 (d) 385
23. The difference of the squares of two consecutive even integers is divisible by which of the following integers?
 (a) 3 (b) 4
 (c) 6 (d) 7

24. If the number 91876 * 2 is completely divisible by 8, then the smallest whole number in place of * will be:
 (a) 1 (b) 2
 (c) 3 (d) 4
25. If 60% of $\frac{3}{5}$ of a number is 36, then the number is:
 (a) 80 (b) 100
 (c) 75 (d) 90
26. What least value should be assigned to * so that the number 451*603 is exactly divisible by 9?
 (a) 2 (b) 5
 (c) 8 (d) 7
27. How many three digits numbers are divisible by 6 in all?
 (a) 149 (b) 150
 (c) 151 (d) 166
28. The largest natural number which exactly divides the product of any four consecutive natural numbers is:
 (a) 6 (b) 12
 (c) 24 (d) 120
29. The sum of all two digit numbers divisible by 5 is:
 (a) 1035 (b) 1245
 (c) 1230 (d) 945
30. If the number 653 .xy is divisible by 90, then $(x + y) = ?$
 (a) 2 (b) 3
 (c) 4 (d) 6

Answer Key

1 (c)	2 (b)	3 (a)	4 (a)	5 (a)	6 (d)
7 (d)	8 (b)	9 (b)	10 (b)	11 (a)	12 (d)
13 (b)	14 (c)	15 (d)	16 (a)	17 (c)	18 (b)
19 (d)	20 (c)	21 (b)	22 (d)	23 (b)	24 (c)
25 (b)	26 (c)	27 (b)	28 (c)	29 (d)	30 (c)

Explanatory Notes

1. (c)
 Unit digit in $3^4 = 1 \Rightarrow$ Unit digit in $(3^4)^{16} = 1$
 \therefore Unit digit in $3^{65} =$ Unit digit in $[(3^4)^{16} \times 3] = (1 \times 3) = 3$
 Unit digit in $6^{59} = 6$
 Unit digit in 7 \Rightarrow Unit digit in $(7^4)^{17}$ is 1
 Unit digit in $7^{71} =$ Unit digit in $[(7^4)^{17} \times 7^3] = (1 \times 3) = 3$
 \therefore Required digit = Unit digit in $(3 \times 6 \times 3) = 4$
2. (b)
 Number = $269 \times 68 + 0 = 18292$

$$\begin{array}{r} 67 \overline{)18292(237} \\ \underline{134} \\ 489 \\ \underline{469} \\ 202 \\ \underline{201} \\ 1 \end{array}$$

 Therefore, required remainder = 1
3. (a)
 By hit and trial, we find that
 $47619 \times 7 = 333333$
4. (a)
 $80 = 2 \times 5 \times 8$
 Since 653 .xy is divisible by 2 and 5 both, so $y = 0$
 Now, 653x0 is divisible by 8, so 3x0 should be divisible by 8
 This happens when $x = 2$ or 6
 $\therefore x + y = (2 + 0) = 2$ [or] $x + y = (6 + 0) = 6$
5. (a)
 Here $a = 3$ and $r = \frac{6}{3} = 2$. Let the number of terms be n .
 Then, $t_n = 384$
 $\Rightarrow ar^{n-1} = 384$
 $\Rightarrow 3 \times 2^{n-1} = 384$
 $\Rightarrow 2^{n-1} = 128 = 2^7$
 $\Rightarrow n - 1 = 7$

$$\Rightarrow n = 8$$

\therefore Number of terms = 8

6. (d)

By hit and trial, we put $x = 5$ and $y = 1$ so that $(3x + 7y) = (3 \times 5 + 7 \times 1) = 22$, which is divisible by 11

$\therefore (4x + 6y) = (4 \times 5 + 6 \times 1) = 26$, which is not divisible by 11;

$(x + y + 4) = (5 + 1 + 4) = 10$, which is not divisible by 11;

$(9x + 4y) = (9 \times 5 + 4 \times 1) = 49$, which is not divisible by 11;

$(4x - 9y) = (4 \times 5 - 9 \times 1) = 11$, which is divisible by 11

7. (d)

Number = (12×35)

Correct Quotient = $420 \div 21 = 20$

8. (b)

This is a G.P. in which $a = 2$, $r = \frac{2^2}{2} = 2$ and $n = 9$

$$\therefore S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{2 \times (2^9 - 1)}{(2 - 1)}$$

$$= 2 \times (512 - 1) = 2 \times 511 = 1022$$

9. (b)

This is an A.P. in which $a = 6$, $d = 6$ and $S_n = 1800$

Then, $n/2 [2a + (n - 1)d] = 1800$

$$\Rightarrow n/2 [2 \times 6 + (n - 1) \times 6] = 1800$$

$$\Rightarrow 3n(n + 1) = 1800$$

$$\Rightarrow n(n + 1) = 600$$

$$\Rightarrow n^2 + n - 600 = 0$$

$$\Rightarrow n^2 + 25n - 24n - 600 = 0$$

$$\Rightarrow n(n + 25) - 24(n + 25) = 0$$

$$\Rightarrow (n + 25)(n - 24) = 0$$

$$\Rightarrow n = 24$$

Number of terms = 24

10. (b)

Unit digit in $7^{95} = \text{Unit digit in } [(7^4)^{23} \times 7^3]$

$= \text{Unit digit in } [(\text{Unit digit in } (2401))^{23} \times (343)]$

$= \text{Unit digit in } (1^{23} \times 343)$

$= \text{Unit digit in } (343) = 3$

Unit digit in $3^{58} = \text{Unit digit in } [(3^4)^{14} \times 3^2]$

$= \text{Unit digit in } [\text{Unit digit in } (81)^{14} \times 3^2]$

$= \text{Unit digit in } [(1)^{14} \times 3^2]$

$= \text{Unit digit in } (1 \times 9)$

$= \text{Unit digit in } (9) = 9$

Unit digit in $(7^{95} - 3^{58})$

$= \text{Unit digit in } (343 - 9)$

$= \text{Unit digit in } (334) = 4$

So, Option B is the answer.

11. (a)

For every natural number n , $(x^n - a^n)$ is completely divisible by $(x - a)$.

12. (d)

$$\begin{aligned} (3^{25} + 3^{26} + 3^{27} + 3^{28}) &= 3^{25} \times (1 + 3 + 3^2 + 3^3) = 3^{25} \times 40 \\ &= 3^{24} \times 3 \times 4 \times 10 \\ &= (3^{24} \times 4 \times 30), \text{ which is divisible by } 30 \end{aligned}$$

13. (b)

Let $n = 4q + 3$. Then $2n = 8q + 6 = 4(2q + 1) + 2$

Thus, when $2n$ is divided by 4, the remainder is 2

14. (c)

Let the given number be $476.xy0$

Then $(4 + 7 + 6 + x + y + 0) = (17 + x + y)$ must be divisible by 3.

And, $(0 + x + 7) - (y + 6 + 4) = (x - y - 3)$ must be either 0 or 11.

$$x - y - 3 = 0$$

$$\Rightarrow y = x - 3$$

$$(17 + x + y) = (17 + x + x - 3) = (2x + 14)$$

$$\Rightarrow x = 2 \text{ or } x = 8$$

$$\therefore x = 8 \text{ and } y = 5$$

15. (d)

$$\text{Given sum} = 9 + \frac{3}{4} + 7 + \frac{2}{17} - \left(9 + \frac{1}{15}\right)$$

$$= (9 + 7 - 9) + \left(\frac{3}{4} + \frac{2}{17} - \frac{1}{15}\right)$$

$$= 7 + \frac{765 + 120 - 68}{1020}$$

$$= 7 + \frac{817}{1020}$$

16. (a)

Clearly, $(2272 - 875) = 1397$, is exactly divisible by N .

Now, $1397 = 11 \times 127$

\therefore The required 3-digit number is 127, the sum of whose digits is 10.

17. (c)

$$987 = 3 \times 7 \times 47$$

So, the required number must be divisible by each one of 3, 7, 47

$553681 \rightarrow$ (Sum of digits = 28, not divisible by 3)

$555181 \rightarrow$ (Sum of digits = 25, not divisible by 3)

555681 is divisible by 3, 7, 47

18. (b)

When n is odd, $(x^n + a^n)$ is always divisible by $(x + a)$.

\therefore Each one of $(47^{43} + 43^{43})$ and $(47^{47} + 43^{43})$ is divisible by $(47 + 43)$

19. (d)

$$\text{Divisor} = (5 \times 46) = 230$$

$$\therefore 10 \times \text{Quotient} = 230$$

$$\Rightarrow \text{Quotient} = 230/10 = 23$$

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

$$= (230 \times 23) + 46$$

$$= 5290 + 46$$

$$= 5336$$

20. (c)

$$\text{Given Exp.} = [(a + b)^2 - 4ab], \text{ where } a = 476 \text{ and } b = 424$$

$$= [(476 + 424)^2 - 4 \times 476 \times 424]$$

$$= [(900)^2 - 807296]$$

$$= 810000 - 807296$$

$$= 2704$$

21. (b)

$$(4^{61} + 4^{62} + 4^{63} + 4^{64})$$

$$= 4^{61} \times (1 + 4 + 4^2 + 4^3)$$

$$= 4^{61} \times 85$$

$$= 4^{60} \times (4 \times 85)$$

$$= (4^{60} \times 340), \text{ which is divisible by } 10$$

22. (d)

$$\text{We know that } (1^2 + 2^2 + 3^2 + \dots + n^2) = \frac{1}{6}n(n+1)(2n+1)$$

$$\text{Putting } n = 10, \text{ required sum} = \frac{1}{6} \times 10 \times 11 \times 21 = 385$$

23. (b)

Let the two consecutive even integers be $2n$ and $(2n + 2)$.
Then,

$$(2n + 2)^2 = (2n + 2 + 2n)(2n + 2 - 2n)$$

$$= 2(4n + 2)$$

$$= 4(2n + 1), \text{ which is divisible by } 4$$

24. (c)

Then number 6×2 must be divisible by 8.

$$\therefore x = 3, \text{ as } 632 \text{ is divisible } 8.$$

25. (b)

Let the number be X . Then

$$60\% \text{ of } \frac{3}{5} \text{ of } X = 36$$

$$\Rightarrow \left(\frac{60}{100}\right) \times \left(\frac{3}{5}\right) \times X = 36$$

$$\Rightarrow X \left(\frac{36 \times 25}{9}\right) = 100$$

$$\therefore \text{Required number} = 100$$

26. (c)

A number is divisible by 9, when the sum of digits is divisible by 9. Sum of digits in the number $(451*603)$ is 19. Now next multiple of 9 after 18 is 27. Therefore, value 8, $(27 - 19)$ should be assigned to *.

27. (b)

First three digit numbers divisible by 6 is 102. Each subsequent number with a difference of 6 is divisible by 6.

Therefore, required numbers are 102, 108, 114, ..., 996.

This is an AP with $a = 102$ and $d = 6$

$$996 = 102 + (n - 1)6$$

$$\text{Thus, } n = 150$$

28. (c)

$$\text{The required number is } 1 \times 2 \times 3 \times 4 = 24$$

29. (d)

Required numbers are 10, 15, 20, 25, ..., 95

This is an A.P. in which $a = 10$, $d = 5$ and $l = 95$

$$t_n = 95 \Rightarrow a + (n - 1)d = 95$$

$$\Rightarrow 10 + (n - 1) \times 5 = 95$$

$$\Rightarrow (n - 1) \times 5 = 85$$

$$\Rightarrow (n - 1) = 17$$

$$\Rightarrow n = 18$$

$$\therefore \text{Required Sum} = \frac{n}{2}(a + l) = \frac{18}{2} \times (10 + 95) \\ = (9 \times 105) = 945$$

30. (c)

$$90 = 10 \times 9$$

Clearly, $653xy$ is divisible by 10, so $y = 0$

Now, $653x0$ is divisible by 9

So, $(6 + 5 + 3 + x + 0) = (14 + x)$ is divisible by 9

$$\text{So, } x = 4$$

$$\text{Hence, } (x + y) = (4 + 0) = 4$$

H.C.F. AND L.C.M.

Learning Objectives

After going through this chapter, you will be able to learn:

- Concept of factors and multiples
- Different methods of finding HCF and LCM of a given numbers
- Various types of questions asked on HCF and LCM

Facts and Formulae

Factors and Multiples

If a number 'a' divides another number 'b' exactly, we say that 'a' is factor of 'b'. In this case, 'b' is called a multiple of 'a'.

Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.)

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers:

- (i) **Factorization Method:** Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.
- (ii) **Division Method:** Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Finding the H.C.F. of more than two numbers: Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

Least Common Multiple (L.C.M.)

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

- (i) **Factorization Method of Finding L.C.M.:** Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors,
- (ii) **Common Division Method (Short-cut Method) of Finding L.C.M.:** Arrange the given numbers in a row in any order. Divide them by a number which divides exactly

at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of divisors and undivided numbers is the required L.C.M. of the given numbers,

Product of two numbers = Product of their H.C.F. and L.C.M.

Co-Primes

Two numbers are said to be co-primes if their H.C.F. is 1.

H.C.F. and L.C.M. of Decimal Fractions

In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

Comparison of Fractions

Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

H.C.F. and L.C.M. of Fractions

$$\text{H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$

$$\text{L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$

Solved Examples

Example 1:

Find the largest number of four digits exactly divisible by 12, 15, 18 and 27.

Solution:

The largest number of four digits is 9999.

Required number must be divisible by L.C.M. of 12, 15, 18, 27 i.e. 540.

On dividing 9999 by 540, we get 279 as remainder.

Required number = (9999 – 279) = 9720.

Example 2:

Find the smallest number of five digits exactly divisible by 16, 24, 36 and 54

Solution:

Smallest number of five digits is 10000.

Required number must be divisible by L.C.M. of 16, 24, 36, 54 i.e. 432,

On dividing 10000 by 432, we get 64 as remainder.

Required number = $10000 + (432 - 64) = 10368$

Example 3:

Find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.

Solution:

Here, $(20 - 14) = 6$, $(25 - 19) = 6$, $(35 - 29) = 6$ and $(40 - 34) = 6$.

Required number = (L.C.M. of 20, 25, 35, 40) $- 6 = 1394$

Example 4:

Find the least number which when divided by 5, 6, 7, and 8 leaves a remainder 3, but when divided by 9 leaves no remainder.

Solution:

L.C.M. of 5, 6, 7, 8 = 840

Required number is of the form $840k + 3$

The least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2$.

Required number = $(840 \times 2 + 3) = 1683$

Example 5:

The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 seconds respectively. If they all change simultaneously at 8:20:00 hours, then at what time they again change simultaneously.

Solution:

Interval of change = (L.C.M of 48, 72, 108) sec. = 432sec.

So, the lights will again change simultaneously after every 432 seconds i.e, 7 min. 12 sec

Hence, next simultaneous change will take place at 8:27:12 hrs.

Practice Exercise

- Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together?
(a) 4 (b) 10
(c) 15 (d) 16
- Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is:
(a) 4 (b) 5
(c) 6 (d) 8
- The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:
(a) 9000 (b) 9400
(c) 9600 (d) 9800
- The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:
(a) 101 (b) 107
(c) 111 (d) 185
- Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:
(a) 40 (b) 80
(c) 120 (d) 200
- The G.C.D. of 1.08, 0.36 and 0.9 is:
(a) 0.03 (b) 0.9
(c) 0.18 (d) 0.108
- The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:
(a) 1 (b) 2
(c) 3 (d) 4
- The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:
(a) 74 (b) 94
(c) 184 (d) 364
- The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:
(a) 3 (b) 13
(c) 23 (d) 33
- The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder, is:
(a) 1677 (b) 1683
(c) 2523 (d) 3363
- A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they again at the starting point?
(a) 26 minutes and 18 seconds
(b) 42 minutes and 36 seconds
(c) 45 minutes
(d) 46 minutes and 12 seconds
- The ratio of two numbers is 3: 4 and their H.C.F. is 4. Their L.C.M. is:
(a) 12 (b) 16
(c) 24 (d) 48
- The smallest number which when diminished by 7, is divisible 12, 16, 18, 21 and 28 is:
(a) 1008 (b) 1015
(c) 1022 (d) 1032

14. The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:
 (a) 15 cm (b) 25 cm
 (c) 35 cm (d) 42 cm
15. Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is:
 (a) 75 (b) 81
 (c) 85 (d) 89
16. Which of the following fraction is the largest?
 (a) $\frac{7}{8}$ (b) $\frac{13}{16}$
 (c) $\frac{31}{40}$ (d) $\frac{63}{80}$
17. The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is:
 (a) 504 (b) 536
 (c) 544 (d) 548
18. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is:
 (a) 123 (b) 127
 (c) 235 (d) 305
19. Which of the following has the most number of divisors?
 (a) 99 (b) 101
 (c) 176 (d) 182
20. The L.C.M. of two numbers is 48. The numbers are in the ratio 2: 3. Then the sum of numbers is:
 (a) 28 (b) 32
 (c) 40 (d) 64
21. The H.C.F. of $\frac{9}{10}$, $\frac{12}{25}$, $\frac{18}{35}$ and $\frac{21}{40}$ is:
 (a) $\frac{3}{5}$ (b) $\frac{252}{5}$
 (c) $\frac{3}{1400}$ (d) $\frac{63}{700}$
22. If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:
 (a) $\frac{55}{101}$ (b) $\frac{601}{55}$
 (c) $\frac{11}{120}$ (d) $\frac{120}{11}$
23. The LCM of 5, 8, 12, 20 will not be a multiple of:
 (a) 3 (b) 9
 (c) 8 (d) 5
24. An electric wire is sold in multiples of 1 m and a person requires several lengths of wire, each 85 cm long. To avoid any wastage and minimise labour, he should purchase minimum length of:
 (a) 5 m (b) 17 m
 (c) 85 m (d) 1 m
25. The product of two numbers is 12960 and their HCF is 36. How many pairs of such numbers can be formed?
 (a) 3 (b) 4
 (c) 5 (d) 2

Answer Key

1 (d)	2 (a)	3 (c)	4 (c)	5 (a)	6 (c)	7 (b)
8 (d)	9 (c)	10 (b)	11 (d)	12 (d)	13 (b)	14 (c)
15 (c)	16 (a)	17 (d)	18 (b)	19 (c)	20 (c)	21 (c)
22 (c)	23 (b)	24 (b)	25 (d)			