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Prasoon Kumar

MATHS OLYMPIAD

International Mathematics Olympiad

7

Strictly according to the latest syllabus of Maths Olympiads

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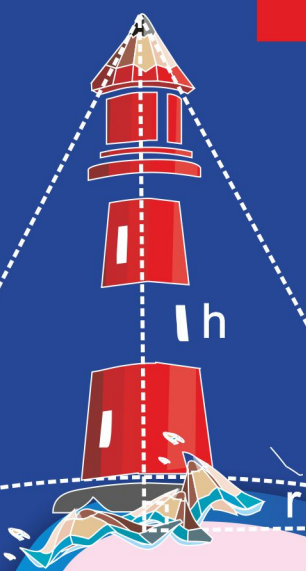

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INTERNATIONAL MATHEMATICS OLYMPIAD 7

Prasoon Kumar

V&S PUBLISHERS

Published by:

V&S PUBLISHERS

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☎ 23240026, 23240027 • *Fax:* 011-23240028
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Branch Office : Mumbai
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ISBN 978-93-505795-8-9

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PUBLISHER'S NOTE

V&S Publishers, after the grand success of a number of academic and general books, is pleased to bring out a series of *Mathematics Olympiad books* under *The Gen X series – generating Xcellence in generation X* – which has been designed to focus on the problems faced by students. In all books the concepts have been explained clearly through various examples, illustrations and diagrams wherever required. Each book has been developed to meet specific needs of students who aspire to get distinctions in the field of mathematics and want to become Olympiad champs at national and international levels.

To go through Maths Olympiad successfully, students need to do thorough study of topics covered in the *Olympiads syllabus and the topics covered in school syllabus as well*. The Olympiads not only tests the subjective knowledge but Reasoning skills also. So students are required to comprehend the depth of concepts and problems and gain experience through practice. The Olympiads check efficiency of candidates in problem solving. These exams are conducted in different stages at regional, national, and international levels. At each stage of the test, the candidate should be fully prepared to go through the exam. Therefore, this exam requires careful attention towards comprehension of concepts, thorough practice, and application of rules and concepts.

While other books in market focus selectively on questions or theory; V&S Maths Olympiad books are rather comprehensive. Each book has been divided into five sections namely *Mathematics, Logical Reasoning, Achiever's section, Subjective section, and Model Papers*. The theory has been explained through solved examples. To enhance problem solving skills of candidates, *Multiple Choice Questions (MCQs)* with detailed solutions are given at the end of each chapter. Two *Mock Test Papers* have been included to understand the pattern of exam. A CD containing Study Chart for systematic preparation, Tips & Tricks to crack Maths Olympiad, Pattern of exam, and links of Previous Years Papers is accompanied with this book. The books are also useful for various competitive exams such as NTSE, NSTSE, and SLSTSE as well.

We wish you all success in the examination and a very bright future in the field of mathematics.

All the best

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SECTION 1 : MATHEMATICAL REASONING

SECTION 2 : LOGICAL REASONING

Part A : Verbal Reasoning

Part B : Non-Verbal Reasoning

Section 1

Mathematical Reasoning

1 Number System

Number system is a system of representing numbers in different forms such as whole number, fraction, decimal etc.

There are different types of numbers, natural numbers, whole numbers, fractions, decimals etc.

Natural Numbers = 1, 2, 3, 4, 5…

Whole Numbers = 0, 1, 2, 3, 4…

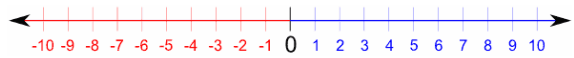
Fractions = 1/2, 1/3, 1/4 … These numbers lie between the two whole numbers.

Decimals = 0.1, 0.2, 0.3 … These numbers are also lie between the two whole numbers.

Integers = Whole numbers along with the negative numbers are called **Integers**.

The numbers …, –4, –3, –2, –1, 0, 1, 2, 3, 4 … etc. are called integer.
1, 2, 3, 4, … are **Positive integer**.
–1, –2, –3, … are **Negative numbers**.

Number Line



Of the two integers represented on the number line, the number on the left is smaller than the number to its right.

Prime Numbers

All the elements in the set of prime numbers are divisible by only two factors, namely 1 and the element itself.

Example: 2, 3, 5, 7, 11, 13.....

Composite Number

Each element in the set of composite numbers has at least one factor other than 1 and the number itself.

Example: 6 have four factors 1, 2, 3 and 6.

Rational Numbers

A rational number of the form $\frac{p}{q}$ or a number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number.

$\frac{1}{2}, \frac{-2}{5}$ are example of rational numbers.

Irrational Numbers

A number which cannot be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called an irrational number.

OR

A number whose decimal expression is non terminating and non recurring is called an irrational number.

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, (22/7) are irrational numbers.

Arithmetic Progression (A.P.)

A sequence is said to be in Arithmetic Progression when it increases or decreases by a constant number. This constant number is called **common difference (c.d.)** of the arithmetic progression.

Examples of Arithmetic Progression are:

- 1, 3, 5, 7… c.d. = 2
- –7, –3, 1, 5, 9 … c.d. = 4

Nth term of an A.P. $(t_n) = a + (n–1)\, d$

Sum of the first n term of an A.P. (S_n)

$= \{2a + (n – 1)\, d\}$

Geometric Progression

A sequence is said to be in Geometric Progression, if the ratio between any two adjacent numbers in the sequence is constant (non zero). This constant is said to be **common ratio (c.r.)**

Examples of Geometric Progression are:

- 1, 2, 4, 8 .....…… c.r. = 2
- 1, 1/2, 1/4, 1/8 …..…… c.r. =1/2

The n-th term of G.P. **(t_n)= $ar^{n–1}$**

Concept of Unit Digits

To understand the concept of Unit Digit, first we have to familiarize with the concept of cyclicity. Cyclicity of any number is about the last digit and how they appear in a certain defined manner. Let's take an example to make this concept clear.

The cyclicity chart of 2 is:

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

In the above chart you have seen that 2 is multiplied every time with its own self, and the last digit changes. On the 4th multiplication, 25 has the same unit digit as 21. This shows us the cyclicity of 2 is 4, that is, after every fourth multiplication, the unit digit will be two.

Cyclicity Table

The cyclicity table for numbers is given below:

Number	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

How did we figure out the above? Multiply and see for yourself. It's a good practice.

Example 1: Now let us use the concept of cyclicity to calculate the unit digit of a number.

What is the unit digit of the expression 445?

Solution: Now we have two methods to solve this; first we choose the best way i.e. cyclicity to solve this.

We know the cyclicity of 4 is 2

Have a look:

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

Here 4 comes again to the end when 4 is raised to the power of 3. So it is clear that the cyclicity of 4 is 2. Now with the cyclicity number i.e. 2, divide the given power i.e. 45/2. In this case the remainder will be 1.

i.e. $4^1 = 4$

So the unit digit in this case is 4.

Example 2: The digit in the unit place of the number 795×358 is:

(a) 7 (b) 2

(c) 6 (d) 4

Solution: The Cyclicity table for 7 is as follows:

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

$7^5 = 16807$

On dividing 95 by 4, the remainder is 3.

Thus, the last digit of 795 is equal to the last digit of 7^3 i.e. 3.

The cyclicity table for 3 is as follows:

$3^1 = 3$

$3^2 = 9$

$33 = 27$
$34 = 81$
$35 = 243$

On dividing 58 by 4, the remainder is 2. Hence, the last digit will be 9.

Therefore, unit's digit of (795×358) is unit's the digit of product of digit at unit's place of 795 and $358 = 3 \times 9 = 27$. Hence option (a) is the answer.

Some Useful Formulas

Sum of all the first n natural numbers $= \frac{n(n+1)}{2}$

- Sum of first n odd numbers $= n^2$
- Sum of first n even numbers $= n\,(n+1)$
- Sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first n natural numbers $= [\frac{n(n+1)}{2}]^2$
- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Example 1: $\frac{854 \times 854 \times 854 - 276 \times 276 \times 276}{854 \times 854 + 854 \times 276 + 276 \times 276} = ?$

(a) 1130 (b) 578
(c) 565 (d) 1156

Solution: Option (b) is correct.

Explanation: Given Exp. $= \frac{(a^3 - b^3)}{(a^2 + ab + b^2)} = (a - b) = (854 - 276) = 578$

Example 2: It is being given that $(2^{32} + 1)$ is completely divisible by a whole number. Which of the following numbers is completely divisible by this number?

(a) $(2^{16} + 1)$ (b) $(2^{16} - 1)$
(c) (7×2^{23}) (d) $(2^{96} + 1)$

Solution: Option (d) is correct.

Explanation: Let $2^{32} = x$. Then, $(2^{32} + 1) = (x + 1)$.

Let $(x + 1)$ be completely divisible by the natural number N. Then,

$(2^{96} + 1) = [(2^{32})^3 + 1] = (x^3 + 1) = (x + 1)(x^2 - x + 1)$, which is completely divisible by N, since $(x + 1)$ is divisible by N.

Example 3: The sum of first 45 natural numbers is:

(a) 1035 (b) 1280

(c) 2070 (d) 2140

Solution: Option (a) is correct.

Explanation: Let $S_n = (1 + 2 + 3 + ... + 45)$

This is an A.P. in which a = 1, d = 1, n = 45 and l = 45

$\therefore S_n = n/2(a + l) = 45/2 \times (1 + 45) = (45 \times 23) = 1035$

Required sum = 1035.

Example 4: What is the unit digit in $\{(6374)^{1793} \times (625)^{317} \times (341^{491})\}$?

(a) 0 (b) 2

(c) 3 (d) 5

Solution: Option (a) is correct.

Explanation: Unit digit in $(6374)^{1793}$ = Unit digit in $(4)^{1793}$

= Unit digit in $[(4^2)^{896} \times 4]$

= Unit digit in $(6 \times 4) = 4$

Unit digit in $(625)^{317}$ = Unit digit in $(5)^{317} = 5$

Unit digit in $(341)^{491}$ = Unit digit in $(1)^{491} = 1$

Required digit = Unit digit in $(4 \times 5 \times 1) = 0$.

Multiple Choice Questions

1. Which of the following is the least part if 128 is divided into four parts proportional to 4, 7, 9 and 12.
   (a) 2^2 (b) 2^3
   (c) 3×2^2 (d) 4^2
2. Select the number which is not prime.
   (a) 131 (b) 181
   (c) 191 (d) 201
3. What is the value of the given expression: $(125 \times 5^6) = ?$
   (a) $5^{3\times6}$ (b) 5^{3+6}
   (c) 5^{3-6} (d) $5^{3\div6}$
4. What smallest number must be added to 6036, so that the sum is completely divisible by 14?
   (a) 12 (b) 17
   (c) 21 (d) 23
5. If there are two odd numbers say x and y, then which of the following expression is even?
   (a) $x + y$ (b) $x + y + 1$
   (c) xy (d) $xy + 2$
6. What is the value of given number sentence?
   $144 \times 36^2 \div 432 = ?$
   (a) 432 (b) 342
   (c) 728 (d) 278
7. Solve the following:
   $43729 \times 999 = ?$
   (a) 43685271 (b) 45685237
   (c) 56358973 (d) 63598742
8. Which of the following is the smallest whole number if the number 418 * 342 is completely divisible by 3?
   (a) 0 (b) 1
   (c) 2 (d) 3
9. Choose the correct number that can replace the question mark (?).
   (?) – 18637 – 23994 = 9977
   (a) 53650 (b) 57760
   (c) 52608 (d) 51560
10. Which of the following is the largest 3-digit prime number?
    (a) 991 (b) 993
    (c) 997 (d) 999
11. Choose the number which is not prime number?
    (a) 617 (b) 627
    (c) 677 (d) 691
12. How many prime numbers exist between 400 and 500?
    (a) 12 (b) 14
    (c) 17 (d) 20
13. The sum of 65 natural number is:
    (a) 2145 (b) 2415
    (c) 1245 (d) 2345
14. How many prime numbers are less than 40?
    (a) 10 (b) 11
    (c) 12 (d) 13
15. The algebraic expression $7^{3n} - 4^{2n}$ has a factor:
    (a) 300 (b) 110
    (c) 327 (d) 427
16. Which of the following rational numbers should be subtracted from $\frac{4}{5}$ to get 0.5?
    (a) 0.8 (b) 0.3
    (c) 1.3 (d) 0.13
17. The additive inverse of 7.22 is added to the additive identity of 3.6. What is the sum obtained?
    (a) 10.82 (b) –7.22
    (c) 3.62 (d) –10.82
18. The multiplicative inverse of $-\frac{2}{15}$ is added to the additive inverse of $-5\frac{1}{2}$. Find the sum.
    (a) –2 (b) 1
    (c) 2 (d) 3

19. $\frac{1}{5}$ is first taken away from 1.5 and then $\frac{6}{5}$ is added to the result. Which of the following is the new rational number obtained?
(a) 3.5 (b) 1.2
(c) 5.2 (d) 2.5

20. The rational number 3.78 is increased by $\frac{1}{5}$ and then $2\frac{1}{4}$ is subtracted from the result. Find the new rational number.
(a) 1.95 (b) 2.25
(c) 9.55 (d) 2.15

21. Which of the following is the unit digit in $\{(6374)^{1793} \times (625)^{317} \times (341^{491})\}$?
(a) 0 (b) 2
(c) 3 (d) 5

22. The last unit digit in the expansion of given expression $71^n - 1$, where n is any positive integer is:
(a) 0 (b) 1
(c) –1 (d) 71

23. Find the value of:
$$\frac{753\times753+247\times247-753\times247}{753\times753\times753+247\times247\times247} = ?$$
(a) $\frac{1}{1000}$ (b) $\frac{3}{1000}$
(c) $\frac{1}{1100}$ (d) $\frac{1}{787}$

24. What is the value of given number expression?
107 x 107 + 93 × 93 = ?
(a) 15378 (b) 16921
(c) 20098 (d) 21368

25. Which of the following is the remainder when we divide $(67^{67} + 67)$ by 68?
(a) 61 (b) 63
(c) 66 (d) 69

26. Find the sum of first 5 prime numbers.
(a) 12 (b) 17
(c) 25 (d) 28

27. Replace question mark with the suitable answer:
$(12345679 \times 2 \times 6^2) = ?$
(a) 880888 (b) 888888888
(c) 80808080 (d) 888888880

28. Find the value of n if $(64)^2 - (36)^2 = 20 \times n$.
(a) 70 (b) 110
(c) 120 (d) 140

29. Which of the following is the correct value of given number expression:
$(2^2 + 4^2 + 6^2 + ... + 20^2) = ?$
(a) 77 × 10 (b) 231 × 5
(c) 2 × 77 × 10 (d) 385 × 385

30. What will be the sum of first forty five counting numbers?
(a) 207 × 5 (b) 207 × 10
(c) 207 × 3 × 5 (d) 207 × 2

31. What will be the sum of even numbers between 1 and 21 is?
(a) 110 (b) 120
(c) 230 (d) 240

32. What is the value obtained if we multiply 2056 and 987?
(a) 1936372 (b) 2029272
(c) 1896172 (d) 2022972

33. If two-third of three-fourth of a number is 30, then three – fifth of that number is:
(a) 25 (b) 35
(c) 36 (d) $\frac{1}{30}$

34. What is the value of third integer if 3 times the first of 3 odd consecutive integers is three more than twice the third?
(a) 3^3 (b) 3^2
(c) 3 × 5 (d) 3 × 15

35. Find the value of given number expression:
397 × 397 + 104 × 104 + 2 × 397 × 104 = ?
(a) 251001 (b) 205010
(c) 205001 (d) 250010

36. Which of the following value is true in place of question mark?
(35423 + 7164 + 41720) – (317 × 89) = ?
(a) 56904 (b) 65904
(c) 50694 (d) 56094

37. $(a^n - b^n)$ is completely divisible by $(a - b)$, when
    (a) n is any natural number
    (b) n is composite number
    (c) n is a negative integer
    (d) n is prime

38. Find the divisor by which the given number expression $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ is completely divided.
    (a) 12 (b) 16
    (c) 26 (d) 30

39. When we divide a number say x by 6 it leaves a remainder 3. When we divide the square of the same number by 6, what will be the remainder?
    (a) 0 (b) 3
    (c) 6 (d) 9

40. Which of the following is the remainder when we divide 17^{200} by 18?
    (a) 1^2 (b) 2^2
    (c) 3^2 (d) 4^2

41. What will be the value of $(1^2 + 2^2 + 3^2 + ... + 10^2) = ?$
    (a) 305 (b) 325
    (c) 375 (d) 385

42. What will be the sum of $(51 + 52 + 53 + ... + 100) = ?$
    (a) 1515 (b) 2025
    (c) 3535 (d) 3775

43. What will be the last digit in $(7^{95} - 3^{58})$?
    (a) 0 (b) 1
    (b) 4 (d) 6

44. Find the value of $1904 \times 1904 = ?$
    (a) 3525216 (b) 3625216
    (c) 3425226 (d) 2625218

45. The least six digit number completely divisible by 111 is:
    (a) 100000 (b) 110000
    (c) 100011 (d) 111000

46. Find the number of terms in the given G.P. series 3, 6, 12, 24... 384.
    (a) 6 (b) 7
    (c) 8 (d) 9

47. What will be the largest 5 digit number completely divisible by 91?
    (a) 99824 (c) 99918
    (b) 99924 (d) None of these

48. When we multiply a certain number by 7, we obtain product whose each digit is 3. What will be that number?
    (a) 47619 (b) 48619
    (c) 47819 (d) 47689

49. If 25% of 2/5 of a certain number is 125 then the required number is:
    (a) 250 (b) 750
    (c) 1000 (d) 1250

50. Which of the following cannot be the square of counting number?
    (a) 32761 (b) 42437
    (c) 81225 (d) 20164

Answer Key

1. (d)	2. (d)	3. (b)	4. (a)	5. (a)	6. (a)	7. (a)	8. (c)
9. (c)	10. (c)	11. (b)	12. (c)	13. (a)	14. (c)	15. (c)	16. (b)
17. (b)	18. (c)	19. (d)	20. (a)	21. (a)	22. (a)	23. (a)	24. (c)
25. (c)	26. (d)	27. (b)	28. (d)	29. (c)	30. (a)	31. (a)	32. (b)
33. (c)	34. (c)	35. (a)	36. (d)	37. (a)	38. (d)	39. (b)	40. (a)
41. (d)	42. (d)	43. (c)	44. (b)	45. (c)	46. (c)	47. (b)	48. (a)
49. (d)	50. (b)						

Hints *and* Solutions

1. **(d)**
Numbers proportional to 4, 7, 9 and 12 are $4x$,$7x$, $9x$, $12x$ respectively.
$4x + 7x + 9x + 12x = 128$
$32x = 128$
$x = 4$
$4x = 16$

2. **(d)**
201 is divisible by 3. So, it is not a prime number.

3. **(b)**
$(125 \times 5^6) = 5^3 \times 5^6 = 5^{3+6}$

4. **(a)**
14) 6036 (431
56

43
42

16
14

2
Required number = (14 – 2)
= 12

5. **(a)**
The sum of two odd numbers is even. So, $x + y$ is even.

6. **(a)**
$144 \times 36^2 \div 432 = 144 \times 1296 \div 432$
$= 144 \times 3 = 432$

7. **(a)**
$43729 \times 9999 = 43729 \times (1000 - 1)$
$= 43729 \times 1000 - 43729 \times 1$
$= 43729000 - 43729$
$= 43685271$

8. **(c)**
Sum of digits = $(4 + 1 + 8 + x + 3 + 4 + 2) = (22 + x)$, which must be divisible by 3.
$\therefore x = 2$

9. **(c)**
$(?) - 18637 - 23994 = 9977$
$(?) = 9977 + 18637 + 23994$
$= 52608$

10. **(c)**
997 is the largest 3- digit prime number.

11. **(b)**
627 is divisible by 3. Hence it is not a prime number.

12. **(c)**
Prime numbers between 400 and 500 are:
401 409 419 421 431 433 439 443 449 457 461 463 467 479 487 491 499

13. **(a)**

Sum of all the first n natural numbers

$$= \frac{n(n+1)}{2}$$

$$= \frac{65(65+1)}{2} = 65 \times 33 = 2145$$

14. **(c)**

Prime numbers less than 40 are:

2 3 5 7 11 13 17 19 23 29 31 37

15. **(c)**

$7^{3n} - 4^{2n} = (7^3)^n - (4^2)^n$

$= (343)^n - (16)^n$

Thus 343 – 16 is a factor i.e. 327 is a factor of $7^{3n} - 4^{2n}$

16. **(b)**

$\frac{4}{5} = 0.8$

Hence, required number = 0.8 – 0.50

= 0.3

17. **(b)**

Additive inverse of 7.22 = – 7.22

Additive identity of 3.6 = 0

Hence, sum of – 7.22 and 0 = – 7.22

18. **(c)**

Multiplicative inverse of $-\frac{2}{15} = \frac{-15}{2}$

Additive inverse of $-5\frac{1}{2} = +5\frac{1}{2}$

Hence, sum of $\frac{-15}{2}$ and $+5\frac{1}{2} = 2$

19. **(d)**

Take away $\frac{1}{5}$ from 1.5 = $1.5 - \frac{1}{5}$

$$= \frac{15}{10} - \frac{1}{5}$$

$$= \frac{15-2}{10} = \frac{13}{10}$$

Add $\frac{6}{5}$ to the result = $\frac{13}{10} + \frac{6}{5}$

= 2.5

20. **(a)**

Sum of $5.25 + \frac{1}{2} = 5.75$

New rational number = $5.75 - 3\frac{4}{5}$

= 5.75 – 3.8

= 1.95

21. **(a)**

Unit digit in $(6374)^{1793}$ = Unit digit in $(4)^{1793}$

= Unit digit in $[(42)^{896} \times 4]$

= Unit digit in $(6 \times 4) = 4$

Unit digit in $(625)^{317}$ = Unit digit in $(5)^{317} = 5$

Unit digit in $(341)^{491}$ = Unit digit in $(1)^{491} = 1$

Required digit = Unit digit in $(4 \times 5 \times 1) = 0$.

22. **(a)**

The unit digit in the 71^n for any value of n is 1.

Thus the unit digit is $71^n - 1$ is 0.

23. **(a)**

Given Expression =

$$\frac{(a^2+b^2-ab)}{a^3+b^3}\frac{1}{(a+b)} = \frac{1}{753+247} = \frac{1}{1000}$$

24. **(c)**

$107 \times 107 + 93 \times 93 = (107)^2 + (93)^2$

$= (100 + 7)^2 + (100 - 7)^2$

$= 2 \times [(100)^2 + 7^2]$ [Ref: $(a + b)^2 + (a - b)^2$

$= 2(a^2 + b^2)$]

= 20098

25. **(c)**

$(x^n + 1)$ will be divisible by $(x + 1)$ only when n is odd.

$(67^{67} + 1)$ will be divisible by (67 + 1)

$(67^{67} + 1) + 66$, when divided by 68 will give 66 as remainder.

26. **(d)**

Required sum = (2 + 3 + 5 + 7 + 11) = 28.

27. **(b)**

$(12345679 \times 2 \times 6^2)$ = 12345679 × 72

= 12345679 × (70 +2)

= 12345679 × 70 + 12345679 × 2

= 864197530 + 24691358

= 888888888

28. **(d)**

$20 \times n = (64 + 36)(64 - 36) = 100 \times 28$

$\therefore\ n = \frac{100 \times 28}{20} = 140$

29. **(c)**

$(2^2 + 4^2 + 6^2 + ... + 20^2)$

$= (1 \times 2)^2 + (2 \times 2)^2 + (2 \times 3)^2 + ... + (2 \times 10)^2$

$= (2^2 \times 1^2) + (2^2 \times 2^2) + (2^2 \times 3^2) + ...$

$+ (2^2 \times 10^2)$

$= 2^2 \times [1^2 + 2^2 + 3^2 + ... + 10^2]$

[Formula: $(1^2 + 2^2 + 3^2 + ... + n^2)$

$= \frac{1}{6} n (n+1) (2n+1)]$

$= (4 \times \frac{1}{6} \times 10 \times 11 \times 21)$

$= (4 \times 5 \times 77)$

$= 2 \times 77 \times 10$

30. **(a)**

Let $S_n = (1 + 2 + 3 + ... + 45)$

This is an A.P. in which a = 1, d = 1, n = 45 and l = 45

$\therefore S_n = \frac{n}{2}(a + l) = \frac{45}{2}(1 + 45) = 45 \times 23 = 207 \times 5$

Required sum = 207 × 5.

31. **(a)**

Let $S_n = (2 + 4 + 6 + ... + 20)$. This is an A.P. in which a = 2, d = 2 and l = 20

Let the number of terms be n. Then,

$a + (n - 1)d = 20$

$2 + (n - 1) \times 2 = 20$

$n = 10.$

$\therefore S_n = \frac{n}{2}(a + l) = \frac{10}{2}(2 + 20) = 5 \times 22 = 110$

32. **(b)**

$2056 \times 987 = 2056 \times (1000 - 13)$

$= 2056 \times 1000 - 2056 \times 13$

$= 2056000 - 26728$

$= 2029272$

33. **(c)**

Let the number be *x*.

Then, $\frac{2}{3} \times \frac{3}{4} \times x = 30$

$x = 60$

So, required number = $60 \times \frac{3}{5} = 36$

34. **(c)**

Let the three integers be *x*, *x* + 2 and *x* + 4.

Then, $3x = 2(x + 4) + 3$

$x = 11$

Third integer = $x + 4 = 15$

35. **(a)**

Given number expression = $(397)^2 + (104)^2 + 2 \times 397 \times 104$

$= (397 + 104)^2$

$= (501)^2 = (500 + 1)^2$

$= (500)^2 + (1)^2 + (2 \times 500 \times 1)$

$= 250000 + 1 + 1000$

$= 251001$

36. **(d)**

$(35423 + 7164 + 41720) - (317 \times 89)$

$= (35423 + 7164 + 41720) - (28213)$

$= 84307 - 28213$

$= 56094$

37. **(a)**

For every natural number n, $(a^n - b^n)$ is completely divisible by (a – b).

38. **(d)**

$(3^{25} + 3^{26} + 3^{27} + 3^{28}) = 3^{25} \times (1 + 3 + 3^2 + 3^3)$

$= 3^{25} \times 40$

$= 3^{24} \times 3 \times 4 \times 10$

$= (3^{24} \times 4 \times 30)$, which is divisible by 30.

39. **(b)**

Let x = 6q + 3.

Then, $x^2 = (6q + 3)^2$

$= 36q^2 + 36q + 9$

$= 6(6q^2 + 6q + 1) + 3$

Thus, when x^2 is divided by 6, then remainder = 3.

40. **(a)**

When n is even; $(x^n - a^n)$ is completely divisible by $(x + a)$

$(17^{200} - 1^{200})$ is completely divisible by (17 + 1), i.e., 18.

$(17^{200} - 1)$ is completely divisible by 18.

On dividing 17^{200} by 18, we get 1 as remainder.

41. **(d)**

We know that $(1^2 + 2^2 + 3^2 + ... + n^2)$

$$= \frac{1}{6} n (n+1) (2n+1)$$

Putting n=10, required sum

$$= \frac{1}{6} \times 10(10 + 1)(20 + 1)$$

$$= \frac{1}{6} \times 10 \times 11 \times 21$$

$$= 385$$

42. **(d)**

This is an A.P. in which a = 51, l = 100 and n = 50.

$$\therefore S_n = \frac{n}{2}(a+l) = \frac{50}{2}(51+100) = 25 \times 151 = 3775$$

43. **(c)**

Unit digit in 7^{95} = Unit digit in $[(7^4)^{23} \times 7^3]$

= Unit digit in [(Unit digit in$(2401))^{23}$ $\times (343)]$

= Unit digit in $(1^{23} \times 343)$

= Unit digit in (343)

= 3

Unit digit in 3^{58} = Unit digit in $[(3^4)^{14} \times 3^2]$

= Unit digit in [Unit digit in $(81)^{14} \times 3^2$]

= Unit digit in $[(1)^{14} \times 3^2]$

= Unit digit in (1×9)

= Unit digit in (9)

= 9

Unit digit in $(7^{95} - 3^{58})$

= Unit digit in (343 – 9)

= Unit digit in (334) = 4.

44. **(b)**

$904 \times 1904 = (1904)^2$

$= (1900 + 4)^2$

$= (1900)^2 + (4)^2 + (2 \times 1900 \times 4)$

$= 3610000 + 16 + 15200$

$= 3625216$

45. **(c)**

The least six digit number is 100000.

When 100000 ÷ 111,

Quotient 990 and Remainder = 100

Therefore required number

$=100000 + (111 - 100)$

$= 100000 + 11$

$= 100011$

46. **(c)**

Here a = 3 and r =6/3 =2.

Let the number of terms be n.

Then, $t_n = 384$

$ar^{n-1} = 384$

$3 \times 2^{n-1} = 384$

$2^{n-1} = 128 = 2^7$

$n - 1 = 7$

$n = 8$

Number of terms = 8.

47. **(b)**

Largest 5-digit number = 99999

When 99999÷91, Quotient = 1098 and Remainder = 81

Required number = (99999 – 81)

= 99918.

48. **(a)**

By hit and trial, we find that

$47619 \times 7 = 333333$.

49. **(d)**

Let the required number be n,

Then $25\% \times 2/5 \times n = 125$

By solving the above equation,

$\therefore n = 1250$

50. **(b)**

The square of a natural number never ends in 7.

42437 is not the square of a natural number.

2 Rational Numbers

Natural Numbers

The counting numbers are called natural numbers.

Example: 1, 2, 3 ..... are natural numbers

Whole Numbers

The counting numbers including zero are called whole numbers.

Example: 0, 1, 2, 41, 45 ..... are whole numbers.

Integers

All natural numbers, zero and negatives of natural numbers are called integers.

Example: 5, 7, – 4, –3, 0, 11, 12, etc. are integers.

Rational Numbers

The numbers of the form $\frac{P}{Q}$, where P and Q are integers numbers and $Q \neq 0$ are called rational numbers.

Examples: $\frac{-3}{9}, \frac{-5}{7}, \frac{1}{3}, \frac{2}{5}$, 0, etc

Positive Rational Numbers

A rational number is called positive national number if its numerator or denominator both have same sign.

Examples: $\frac{2}{5}, \frac{-3}{-8}, \frac{-16}{-15}$, etc.

Negative Rational Numbers

A rational number whose numerator and denominator have opposite sign.

Example: $\frac{-2}{3}, \frac{-5}{7}, \frac{7}{-2}$, etc.

(i) If $\frac{a}{b}$ is a rational number and m is a nonzero integer then $\frac{a}{b} = \frac{a \times m}{b \times m}$

(ii) If $\frac{a}{b}$ is a rational number and m is a common divisor then $\frac{a}{b} = \frac{a \div m}{b \div m}$

Example 1: Write two equivalent rational number of $\frac{2}{5}$.

Solution:

$$\frac{2}{5} = \frac{2 \times (-1)}{5 \times (-1)} = \frac{-2}{-5}$$

$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

Example 2: Write $\frac{7}{\cdot}$ with positive denominator.

Solution: $\frac{7}{-12} = \frac{7\times(-1)}{-12\times(-1)} = \frac{-7}{12}$

Example 3: Write $\frac{-6}{13}$ with positive numerator.

Solution: $\frac{-6}{13} = \frac{(-6)(-1)}{(13)(-1)} = \frac{6}{-13}$

Example 4: Express $\frac{-3}{7}$ with denominator 35.

$\frac{-3}{7} = \frac{-3\times5}{7\times5} = \frac{-15}{35}$

Standard Form of a Rational Number: A rational number $\frac{a}{b}$ is said to be in standard form if b is positive and a and b have no common divisor other than 1.

Example 5: Express $\frac{-36}{-54}$ in standard form.

Solution: $\frac{-36}{-54} = \frac{-36\times-1}{-54\times-1} = \frac{36}{54} = \frac{2}{3}$

If $\frac{P}{Q} = \frac{R}{S}$ then $P \times S = Q \times R$

Example 6: Find x if $\frac{-3}{7} = \frac{x}{84}$

Solution: Here, $7\times x = -3 \times 84$

$\Rightarrow x = \frac{-3\times84}{7} = -36$

Example 7: Which of the two rational numbers $\frac{-2}{3}$ and $\frac{-4}{5}$ is greater?

Solution: $\frac{-2}{3}$ $\frac{-4}{5}$

-2×5 3×-4

$-10 > -12$

$\therefore \frac{-2}{3} > \frac{-4}{5}$

Example 8: List five rational numbers between –1 and 0.

Solution: $-1 = \frac{-1\times6}{1-6} = \frac{-6}{6}$

$\frac{-6}{6} < \frac{-5}{6} < \frac{-4}{6} < \frac{-3}{6} < \frac{-2}{6} < \frac{-1}{6} < 0$

Example 9: Add $\frac{7}{-27}+\frac{11}{18}$

Solution: $\frac{7}{-27}+\frac{11}{18}$

$= \frac{-14+33}{54}=\frac{19}{54}$

Example10: Subtract $-\frac{5}{7}$ from $-\frac{2}{3}$.

Solution: $-\frac{2}{3}-\left(-\frac{5}{7}\right)$

$= -\frac{2}{3}+\frac{5}{7}$

$= \frac{-14+15}{21}=\frac{1}{21}$

Example11: What is additive inverse of $\frac{-12}{-17}$?

Solution: Here, $\frac{-12}{-17}=\frac{(-12)\times(-1)}{(-17)\times(-1)}=\frac{12}{17}$

$\therefore$ Additive inverse of $\frac{12}{17}=\frac{-12}{17}$

Example 12: Sum of two rational numbers is $\frac{4}{21}$. If one of them is $\frac{5}{7}$ then what is the other?

Solution: If x is required number then $\frac{5}{7}+x=\frac{4}{21}$

$x = \frac{4}{21}-\frac{5}{7}$

$= \frac{4-15}{21}=\frac{-11}{21}$

Example 13: Simplify $\frac{-9}{16}\times\frac{-64}{45}$

Solution: $\frac{-9}{16}\times\frac{-64}{45}=\frac{(-1)(-4)}{(1)(5)}=\frac{4}{5}$

Reciprocal of a Rational Number

If the product of two rational numbers is 1 then each of them is called the reciprocal of the other.

Example14: Find the reciprocal of $\frac{4}{17}$.

Solution: $\frac{4}{17}\times x=1$

$$\Rightarrow x = 1 \div \frac{4}{17}$$

$$= 1 \times \frac{17}{4} = \frac{17}{4}$$

Division of Two Rational Numbers

If $\frac{a}{b}$ and $\frac{d}{c}$ be two rational numbers such that $\frac{c}{d} \neq 0$ then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Example 15: By what number should $-\frac{8}{15}$ be multiplied to get 24?

Solution: $-\frac{8}{15} \times x = 24$

$$\Rightarrow \quad x = 24 \div -\frac{8}{15}$$

$$= 24 \times -\frac{15}{8} = -45$$

Example16: By what rational number should $-\frac{8}{65}$ to multiplied to get $\frac{5}{26}$?

Solution: $-\frac{8}{65} \times x = \frac{5}{26}$

$$\Rightarrow \quad x = \frac{5}{26} \div -\frac{8}{65}$$

$$= \frac{5}{26} \times \frac{-65}{8} = \frac{-25}{16}$$

Multiple Coice Questions

1. By what number should $\frac{-33}{8}$ be divided to get $\frac{11}{2}$?

   (a) $\frac{3}{4}$ (b) $-\frac{3}{4}$

   (c) $\frac{4}{3}$ (d) $-\frac{4}{3}$

2. The product of two rational numbers is –10. If one of the numbers is 8, what is the other?

   (a) $\frac{4}{5}$ (b) $\frac{5}{4}$

   (c) $-\frac{4}{5}$ (d) $-\frac{5}{4}$

3. The sum of two rational numbers is –7. If one of the numbers is $-\frac{15}{6}$, what is the other?

   (a) $\frac{9}{2}$ (b) $-\frac{9}{2}$

   (c) $\frac{7}{2}$ (d) $-\frac{7}{2}$

4. Find the additive inverse of $\left(\frac{4}{5}+\frac{3}{7}\right)$.

   (a) $\frac{43}{15}$ (b) $-\frac{43}{15}$

   (c) $\frac{15}{43}$ (d) $-\frac{15}{43}$

5. What should be added to $\frac{-7}{8}$ to get $\frac{5}{9}$?

   (a) $\frac{103}{72}$ (b) $-\frac{103}{72}$

   (c) $\frac{72}{103}$ (d) $-\frac{72}{103}$

6. What is the value of $\left(2-\frac{1}{2}-\frac{3}{4}\right)$?

   (a) $\frac{1}{4}$ (b) $\frac{3}{4}$

   (c) $\frac{1}{2}$ (d) $-\frac{3}{4}$

7. What should be added to $\frac{2}{9}$ to get –3?

   (a) $-\frac{29}{9}$ (b) $\frac{23}{9}$

   (c) $\frac{31}{9}$ (d) $-\frac{31}{9}$

8. The sum of two numbers is 12. If one of them is $-\frac{7}{9}$, what is the other?

   (a) $\frac{113}{9}$ (b) $\frac{115}{9}$

   (c) $\frac{117}{9}$ (d) $\frac{103}{9}$

9. What is the simplest form of $\frac{84}{288}$?

   (a) $\frac{7}{12}$ (b) $\frac{7}{48}$

   (c) $\frac{7}{24}$ (d) None of these

10. Which of the following is correct?

    (a) $-\frac{2}{3}>\frac{-4}{3}>\frac{-7}{3}>\frac{-11}{3}$ (b) $\frac{2}{3}>\frac{4}{3}>\frac{7}{3}$

    (c) $\frac{1}{2}<\frac{1}{3}<\frac{1}{4}<\frac{1}{5}$ (d) None of these

11. What is the value of x if $\frac{x}{7}=\frac{63}{18}$?

    (a) $\frac{2}{49}$ (b) $\frac{3}{49}$

    (c) $\frac{49}{2}$ (d) None of these

12. By what rational number $\frac{2}{7}$ is divided to get 5?

(a) $\frac{2}{35}$ (b) $\frac{35}{2}$

(c) $\frac{5}{7}$ (d) $\frac{2}{7}$

13. By what rational number $\frac{3}{13}$ is multiplied to get –12?

(a) –55 (b) –54

(c) –53 (d) –52

14. Which of the following is incorrect?

(a) $\frac{1}{2} > \frac{1}{3} > \frac{1}{4}$ (b) $\frac{2}{5} < \frac{4}{5} < 1$

(c) $\frac{2}{3} > \frac{3}{4} > \frac{7}{8}$ (d) $\frac{\cdot}{\cdot} < - < -$

15. Which rational number is in between 2 and 3?

(a) $\frac{8}{3}$ (b) $\frac{16}{3}$

(c) $\frac{15}{4}$ (d) $\frac{11}{3}$

16. The cost of $4\frac{1}{2}$ metres of cloth is ₹ $98\frac{3}{4}$. What is the cost of cloth per metre?

(a) $\frac{295}{18}$ (b) $\frac{395}{18}$

(c) $\frac{495}{18}$ (d) None of these

17. What is the simplified value of $\left(\frac{3}{55} \times \frac{-33}{18}\right) - \left(\frac{39}{125} \times \frac{-15}{78}\right)$?

(a) $\frac{1}{25}$ (b) $\frac{1}{50}$

(c) $-\frac{1}{50}$ (d) $-\frac{1}{25}$

18. What is the reciprocal of $\left[\frac{2}{3} \div \frac{1}{3} - \frac{1}{2} \times \frac{1}{2}\right]$?

(a) $\frac{4}{7}$ (b) $\frac{5}{7}$

(c) $\frac{3}{7}$ (d) $\frac{-4}{7}$

19. The reciprocal of a rational number is $\frac{-7}{9}$. What is that rational number?

(a) $\frac{-9}{7}$ (b) $\frac{9}{7}$

(c) $\frac{7}{9}$ (d) 1

20. The sum of reciprocals of two rational numbers is $\frac{7}{4}$. If one of the numbers is $\frac{2}{3}$ what is the other?

(a) 2 (b) – 4

(c) 4 (d) None of these

21. Which of the following is improper rational number?

(a) $\frac{1}{7}$ (b) $\frac{2}{3}$

(c) $\frac{4}{13}$ (d) $\frac{7}{2}$

22. Which of the following is equivalent to $\frac{4}{7}$?

(a) $\frac{12}{14}$ (b) $\frac{12}{21}$

(c) $\frac{16}{21}$ (d) $\frac{20}{28}$

23. What is the value of x if $\frac{121}{13} = \frac{x}{104}$?

(a) 948 (b) 968

(c) 978 (d) 988

24. What result will be obtained when the sum of $\frac{65}{12}$ and $\frac{8}{3}$ is divided by their difference?

(a) $\frac{33}{97}$ (b) $\frac{97}{33}$

(c) $\frac{31}{97}$ (d) $\frac{97}{31}$

25. What is multiplicative inverse of $\left(\frac{2}{3}+\frac{3}{4}\right)$?

(a) $\frac{12}{17}$ (b) $-\frac{12}{17}$

(c) 1 (d) 0

26. The cost of 15 articles is ₹ $87\frac{1}{2}$. What is the cost of one article?

(a) ₹$\frac{25}{6}$ (b) ₹$\frac{35}{6}$

(c) ₹$\frac{37}{6}$ (d) None of these

27. A bus is moving at an average speed of $56\frac{2}{3}$ km/hour. How much distance it cover in $3\frac{2}{5}$ hour?

(a) $\frac{578}{3}$ km (b) $\frac{568}{3}$ km

(c) $\frac{548}{3}$ km (d) None of these

28. What should be added to $\left(\frac{-23}{4}+\frac{-13}{8}\right)$ to get1?

(a) $\frac{57}{8}$ (b) $\frac{59}{8}$

(c) $\frac{67}{8}$ (d) None of these

29. How many pieces each of length $3\frac{3}{4}$ m can be cut from a rope of length 60 m?

(a) 14 (b) 15

(c) 16 (d) 17

30. By what rational number should $\frac{-8}{65}$ is multiplied to obtain $\frac{5}{39}$?

(a) $-\frac{5}{24}$ (b) $\frac{25}{24}$

(c) $\frac{5}{24}$ (d) None of these

31. The product of two rational numbers is $\frac{-32}{9}$. If one of them is $\frac{-8}{3}$ what is the other?

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $-\frac{3}{4}$ (d) None of these

32. What rational number should be subtracted from 1 to get $-\frac{3}{7}$?

(a) $\frac{7}{10}$ (b) $\frac{10}{7}$

(c) $\frac{-7}{10}$ (d) $\frac{-10}{7}$