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Prasoon Kumar

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International Mathematics Olympiad

9

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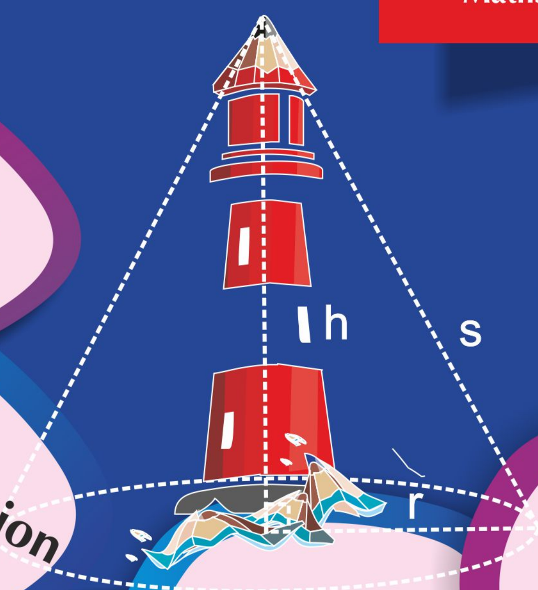

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INTERNATIONAL MATHEMATICS OLYMPIAD

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Regional Office : Hyderabad
5-1-707/1, Brij Bhawan (Beside Central Bank of India Lane)
Bank Street, Koti, Hyderabad - 500 095
☎ 040-24737290
E-mail: vspublishershyd@gmail.com

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☎ 022-23510736
E-mail: vspublishersmum@gmail.com

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Publisher's Note

V&S Publishers, after the grand success of a number of academic and general books, is pleased to bring out a series of *Mathematics Olympiad books* under *The Gen X series – generating Xcellence in generation X* – which has been designed to focus on the problems faced by students. In all books the concepts have been explained clearly through various examples, illustrations and diagrams wherever required. Each book has been developed to meet specific needs of students who aspire to get distinctions in the field of mathematics and want to become Olympiad champs at national and international levels.

To go through Maths Olympiad successfully, students need to do thorough study of topics covered in the *Olympiads syllabus and the topics covered in school syllabus as well*. The Olympiads not only tests the subjective knowledge but Reasoning skills also. So students are required to comprehend the depth of concepts and problems and gain experience through practice. The Olympiads check efficiency of candidates in problem solving. These exams are conducted in different stages at regional, national, and international levels. At each stage of the test, the candidate should be fully prepared to go through the exam. Therefore, this exam requires careful attention towards comprehension of concepts, thorough practice, and application of rules and concepts.

While other books in market focus selectively on questions or theory; V&S Maths Olympiad books are rather comprehensive. Each book has been divided into five sections namely *Mathematics, Logical Reasoning, Achiever's section, Subjective section, and Model Papers*. The theory has been explained through solved examples. To enhance problem solving skills of candidates, *Multiple Choice Questions (MCQs)* with detailed solutions are given at the end of each chapter. Two *Mock Test Papers* have been included to understand the pattern of exam. A CD containing Study Chart for systematic preparation, Tips & Tricks to crack Maths Olympiad, Pattern of exam, and links of Previous Years Papers is accompanied with this book. The books are also useful for various competitive exams such as NTSE, NSTSE, and SLSTSE as well.

We wish you all success in the examination and a very bright future in the field of mathematics.

All the best

Contents

SECTION 1 : MATHEMATICAL REASONING

SECTION 2 : LOGICAL REASONING

Part A : Verbal Reasoning

Part B : Non-Verbal Reasoning

SECTION 3 : ACHIEVER'S SECTION

SECTION 4 : SUBJECTIVE SECTION

SECTION 5 : MODEL PAPERS

Section 1

Mathematical Reasoning

1 Number System

Learning Objective:

In this chapter we shall learn about :

- Natural numbers
- Whole numbers
- Rational numbers
- Irrational numbers and real numbers

Natural Numbers

Counting numbers 1, 2, 3, …… are known as natural numbers.

Thus 1, 2, 3, 4, 5, 6, 7, …… are natural numbers. It is denoted by N. Hence, N = {1, 2, 3, 4, ……}

Whole Numbers

All natural numbers along with zero are called whole numbers. It is denoted by W.

Hence W = {0, 1, 2, 3, 4, ……}

Integers

All natural numbers, zero and negatives of natural numbers form the set integers.

Example: 0, 1, –1, 2, –2, 3, –3, ... etc., are integers

$\therefore$ Natural numbers $\in$ Whole numbers $\in$ Integers

Rational Numbers

The numbers of the form — , where *p*, *q* are integers and $q \neq 0$ are known as rational numbers.

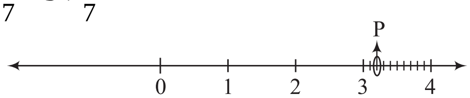
Example: $\frac{-1}{2}, \frac{3}{2}, \frac{7}{9}, \frac{-7}{6}, \frac{8}{9}$ -------- etc., are rational numbers.

Example 1: Write 3 rational numbers equivalent to $\frac{6}{5}$.

Solution: We have $\frac{6}{5} = \frac{6 \times 2}{5 \times 2} = \frac{6 \times 5}{5 \times 5} = \frac{6 \times 3}{5 \times 3} = \frac{12}{10} = \frac{30}{25} = \frac{18}{15}$

Example 2: Represent $3\frac{2}{7}$ on real line.

Solution: We have $3\frac{2}{7} = 3 + \frac{2}{7}$



Divide the portion between 3 and 4 to 7 equal parts and mark the second spot, i.e., *P*.

P will represent $3\frac{2}{7}$ on real line.

Example 3: Insert five rational numbers between 6 and 8.

Solution: $d = \frac{y-x}{n+1} = \frac{8-6}{5+1} = \frac{2}{6} = \frac{1}{3}$

$\therefore$ Five rational numbers between 6 and 8 are $\left(6+\frac{1}{3}\right), \left(6+\frac{2}{3}\right), \left(6+\frac{3}{3}\right), \left(6+\frac{4}{3}\right), \left(6+\frac{5}{3}\right)$

$$= \left(\frac{19}{3}\right), \left(\frac{20}{3}\right), \left(\frac{21}{3}\right), \left(\frac{22}{3}\right), \left(\frac{23}{3}\right)$$

Example 4: Find four rational numbers between — and 1.

Solution: We have $d = \frac{1-\frac{1}{2}}{4+1} = \frac{1}{10}$

$\therefore$ Four rational numbers between $\frac{1}{2}$ and 1 are $\left(\frac{1}{2}+\frac{1}{10}\right), \left(\frac{1}{2}+\frac{2}{10}\right), \left(\frac{1}{2}+\frac{3}{10}\right), \left(\frac{1}{2}+\frac{4}{10}\right)$.

$$= \left(\frac{6}{10}\right), \left(\frac{7}{10}\right), \left(\frac{8}{10}\right), \left(\frac{9}{10}\right)$$

Example 5: Write nine rational numbers between 0 and 3.

Solution: Here $d = \frac{3-0}{9+1} = \frac{3}{10}$

$\therefore$ Nine rational numbers between 0 and 3 are

$$\left(0+\frac{3}{10}\right), \left(0+\frac{6}{10}\right), \left(0+\frac{9}{10}\right), \left(0+\frac{12}{10}\right) \text{ ....,} \left(0+\frac{27}{10}\right)$$

Required rational numbers are $\frac{3}{10}, \frac{6}{10}, \frac{9}{10}, \frac{12}{10}, \frac{15}{10}, \frac{18}{10}, \frac{21}{10}, \frac{24}{10}$ and $\frac{27}{10}$.

Terminating Decimal

Every fraction $\frac{p}{q}$ can be expressed as a decimal if the decimal terminates, i,e, comes to an end then the decimal is said to be terminating.

Example: $\frac{1}{8} = 0.125$, $\frac{1}{4} = 0.25$, $\frac{1}{2} = 0.5$, etc

Repeating (Recurring Decimals)

A decimal in which a digit or a set of digits repeats periodically, is called a repeating or a recurring decimal.

Example: **(i)** $\frac{1}{3} = 0.3333 \text{ ------ } = 0.\overline{3}$ **(ii)** $\frac{15}{7} = 2.\overline{142857}$ **(iii)** $\frac{2}{3} = 0.6666 \text{ ------ } = 0.\overline{6}$

Terminating decimals have their denominators of the form $2^m \times 5^n$, where, m and n are natural numbers or even m, n is (are) zero.

Example 6: Find which of the following rational numbers are terminating decimals, without actual division,

(a) $\frac{5}{30}$ (b) $\frac{12}{125}$ (c) $\frac{11}{500}$

Solution: (a) Given denominator = $30 = 2 \times 5 \times 3$

$\because$ denominator has an extra term than 2 and 5. Therefore, decimal is non-terminating.

(b) $125 = 5 \times 5 \times 5 = 2^0 \times 5^3$

$\therefore$ Decimal is terminating.

(c) $500 = 2 \times 5 \times 5 \times 2 \times 5 = 2^2 \times 5^3$

$\because$ Denominator has 2 and 5 as its factors.

$\therefore$ Decimal is terminating.

Example 7: Express each of the following decimals as a fraction in the simplest form:

(a) $0.\overline{36}$ (b) $0.5\overline{4}$ (c) $0.\overline{324}$ (d) $0.1\overline{23}$

Solution: (a) Let $x = 0.\overline{36} = 0.363636\ldots$ ...(i)

$100\,x = 36.3636$ ...(ii)

Using eq. (i), and eq. (ii)

$$99x = 36$$

$$\Rightarrow \quad x = \frac{36}{99} = \frac{4}{11}$$

(b) Let $x = 0.54444$ ...(i)

$10x = 5.4444$ ...(ii)

$100x = 54.4444$ ...(iii)

Using eq .(iii) and eq. (ii)

$$90x = 49$$

$$\Rightarrow \quad x = \frac{49}{90}$$

(c) $x = 0.324324324$ ...(i)

$1000\,x = 324.324324$ ...(ii)

Using eq. (i) and eq. (ii)

$$999\,x = 324$$

$$x = \frac{324}{999} = \frac{36}{111} = \frac{12}{37}$$

(d) $x = 0.1232323$ ...(i)

$10x = 1.232323$ ...(ii)

$100x = 123.232323$ ...(iii)

$1000x = 123.232323$ ...(iv)

Using eq. (iv) and eq. (ii)

$$9990x = 122$$

$$\Rightarrow \quad x = \frac{122}{9990} = \frac{61}{4995}$$

Irrational Numbers

A number which can neither be expressed as a terminating decimal nor as a repeating decimal , is called an irrational number.

Example: $\sqrt{2}, \sqrt{3}, \sqrt{7}, \sqrt{5}$ etc.

Properties of Irrational Numbers

(a) Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.

(b) Sum of two irrationals can be or cannot be irrational.

Example: $\sqrt{3}+\sqrt{2}$ will be irrational, but

$\left(2-\sqrt{2}\right)+\left(2+\sqrt{3}\right) = 4$, which is rational.

(c) Multiplication of two irrationals need not be irrational. The division of two irrationals also behaves same.

Example:

$\sqrt{2}\times\sqrt{3} = \sqrt{6} \rightarrow$ Irrational

$\sqrt{3}\times\sqrt{3} = 3 \rightarrow$ Rational

$\dfrac{\sqrt{3}}{\sqrt{2}} = \sqrt{\dfrac{3}{2}} \rightarrow$ Irrational

$\dfrac{2\sqrt{3}}{\sqrt{3}} = 2 \rightarrow$ Rational

(d) Any operation between a rational and an irrational number will always result in irrational number.

(e) The square root of all positive numbers is not always irrational, same is for the cube root of positive and negative numbers.

Example:

$\sqrt{3} = 1.732$ .... irrational

$\sqrt{2} = 1.414$ .... irrational

$\sqrt{4} = 2$ .... rational

$\sqrt[3]{8} = 2$ .... rational

Real Numbers

A number whose square is non-negative zero or positive is called real number.

Or

The set of rational and irrational numbers together is called real numbers.

Completeness Property

On number line, each point corresponds to an unique real number.

Density Property

Between any two real numbers, there exist infinitely many real numbers.

Properties of Real Numbers

(i) Closure property of addition and multiplication: The sum or the product of two real numbers will result in a real number.

(ii) Associative law: $a + (b + c) = (a + b) + c$, and $a(bc) = (ab)c$,

where a, b and c are real numbers.

(iii) Commutative law: $a + b = b + a$ and $ab = ba$, where a, b are any real numbers.

(iv) Existence of additive and multiplicative identities:

Additive Identity $\Rightarrow$ $a + 0 = 0 + a = a$

Here 0 is additive identity.

Multiplicative Identity $\Rightarrow$ $a.1 = 1.\ a = a$ for every real number 'a'

Here 1 is multiplicative identity.

(v) Existence of additive and multiplicative inverse:

$(-a)$ is additive inverse of 'a' and $\frac{1}{a}$ is multiplicative inverse of a.

(vi) Distributive laws of multiplication over addition:

$$(a + b)\ c = ac + bc, \text{ and, } a(b + c) = ab + ac$$

where, a, b and c are real numbers.

Example 1: Add $\left(2\sqrt{3}+\sqrt{2}\right)$ and $\left(7\sqrt{2}-\sqrt{3}\right)$.

Solution: We have $\left(2\sqrt{3}+\sqrt{2}\right) + \left(7\sqrt{2}-\sqrt{3}\right) = 8\sqrt{2}+\sqrt{3}$

Example 2: Multiply $\left(5+\sqrt{6}\right)$ and $\left(5-\sqrt{6}\right)$.

Solution: $\left(5+\sqrt{6}\right)\left(5-\sqrt{6}\right) = (5)^2 - \left(\sqrt{6}\right)^2 = 25 - 6 = 19$

Example 3: Simplify $\left(\sqrt{3}+\sqrt{5}\right)^2$.

Solution: $\left(\sqrt{3}+\sqrt{5}\right)\left(\sqrt{3}+\sqrt{5}\right)$

$= \sqrt{3}\ \left(\sqrt{3}+\sqrt{5}\right)+\sqrt{5}\left(\sqrt{3}+\sqrt{5}\right)$

$= 3 + \sqrt{15}+\sqrt{15}+5 = 8 + 2\sqrt{15}$

Rationalisation

The process of correcting an irrational denomination to a rational number by multiplying its numerator and denominator by a suitable number is called rationalisation and the number used is called rationalising factor.

To rationalise the denomination of $\frac{1}{\sqrt{x}+y}$, we multiply it by $\frac{\sqrt{x}-y}{\sqrt{x}-y}$, where x, y are integers.

Example 4: Simplify $\frac{2}{\sqrt{3}}$ by rationalising the denominator.

Solution: $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Law of Radicals

If m and n are rational numbers and a is a positive real number then

(i) $a^m \cdot a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^p \times b^p = (ab)^p$

Example 5: Simplify $\frac{1}{2+\sqrt{3}}$.

Solution: We have $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2} = 2 - \sqrt{3}$

Example 6: Solve $\frac{1}{4-\sqrt{15}}$.

Solution: We have $\frac{1}{4-\sqrt{15}} = \frac{4+\sqrt{15}}{(4)^2 - (\sqrt{15})^2} = 4 + \sqrt{15}$

Example 7: If $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$, then find the value of 'a' and 'b'.

Solution: We have $\frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$

$$= \frac{\left(3+\sqrt{2}\right)^2}{(3)^2 - \left(\sqrt{2}\right)^2} = \frac{9+2+6\sqrt{2}}{7} = \frac{11}{7} + \frac{6\sqrt{2}}{7} = a + b\sqrt{2}$$

$\therefore \quad a = \frac{11}{7}, b = \frac{6}{7}$

Example 8: If $x = 2+\sqrt{3}$, then find the value of $x^2 + \frac{1}{x^2}$.

Solution: Given $x = 2 + \sqrt{3}$

$$x^2 = \left(2+\sqrt{3}\right)^2 = 4 + \left(\sqrt{3}\right)^2 + 4\sqrt{3} = 7 + 4\sqrt{3}$$

$$\therefore \quad \frac{1}{x^2} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{\cdot - \sqrt{\ }}{(\cdot)^2 - \left(\ \sqrt{\ }\right)^2} = \frac{7-4\sqrt{3}}{49-48} = 7 - 4\sqrt{3}$$

$$\therefore \quad x^2 + \frac{1}{x^2} = 7+4\sqrt{3} + \cdot - \sqrt{\ } = 14$$

Example 9: $\dfrac{\cdot}{3\aleph\sqrt{8}}+\dfrac{}{\sqrt{7}\quad\sqrt{6}}-\dfrac{}{\sqrt{8}\quad\sqrt{7}}-\dfrac{1}{\sqrt{6}-\sqrt{5}}+\dfrac{1}{\sqrt{5}-2}=x$ then $x=$?

Solution: $\dfrac{1}{3-\sqrt{8}}=\dfrac{3+\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}=3+\sqrt{8}$

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\sqrt{7}+\sqrt{6},\ \frac{1}{\sqrt{8}-\sqrt{7}}=\sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}}=\sqrt{6}+\sqrt{5}\ ,\ \frac{1}{\sqrt{5}-2}=\sqrt{5}+2$$

Using all these and putting it in expression, we have

$$=3+\sqrt{8}+\sqrt{7}+\sqrt{6}-\left(\sqrt{8}+\sqrt{7}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$=3+\sqrt{8}+\sqrt{7}+\sqrt{6}-\sqrt{8}-\sqrt{7}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$=(3+2)=5$$

Example 10: If $(16)^{\frac{3}{2}}=x$ then what is the value of 'x'?

Solution: Here $x=(16)^{\frac{3}{2}}=\left[(4)^2\right]^{\frac{3}{2}}=(4)^{2\times\frac{3}{2}}=(4)^3=64$

Example 11: Simplify $(125)^{\frac{-1}{3}}$.

Solution: We have $(125)^{\frac{-1}{3}}=\left(\dfrac{1}{125}\right)^{\frac{1}{3}}=\left[\left(\dfrac{1}{5}\right)^3\right]^{\frac{1}{3}}=\left(\dfrac{1}{5}\right)^{3\times\frac{1}{3}}=\dfrac{1}{5}$

Example 12: Simplify $(81)^{\frac{-1}{4}}$.

Solution: $(81)^{\frac{-1}{4}}=\left(\dfrac{1}{81}\right)^{\frac{1}{4}}=\left[\left(\dfrac{1}{3}\right)^4\right]^{\frac{1}{4}}=\left(\dfrac{1}{3}\right)^{4\times\frac{1}{4}}=\dfrac{1}{3}$

Example 13: Simplify $(625)^{0.16}\times(625)^{0.09}$.

Solution: $(625)^{0.16+0.09}=(625)^{0.25}=\left[(5)^4\right]^{0.25}=(5)^{4\times0.25}=(5)^1=5$

Example 14: If $x=7+4\sqrt{3}$, then $x+\dfrac{1}{x}=$?

Solution: Given $x=7+4\sqrt{3}$, $\dfrac{1}{x}\aleph\dfrac{1}{7+4\sqrt{3}}\ \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}\ \dfrac{7-4\sqrt{3}}{(\cdot)^2-(\ \sqrt{\ }\ )^2}=7-4\sqrt{3}$

$\therefore\qquad x+\dfrac{1}{x}=\left(7+4\sqrt{3}\right)+\left(7-4\sqrt{3}\right)=14$

Example 15: Evaluate $\left[(64)^{-2}\right]^{\frac{1}{4}}$.

Solution: $\left((64)^{-2}\right)^{\frac{1}{4}} = (64)^{-2\times\frac{1}{4}} = (64)^{-\frac{1}{2}} = \frac{1}{8}$

Multiple Choice Questions

1. Choose the correct statement :
   (a) Every whole number is a natural number.
   (b) Every integer is a rational number.
   (c) Every integer is a whole number.
   (d) Every rational number is an integer

2. Which of the following number is irrational?
   (a) $\frac{7}{8}$ (b) $\sqrt{\frac{9}{125}}$ (c) $\frac{93}{300}$ (d) $\frac{190}{30}$

3. Which of the following decimal is terminating?
   (a) $\frac{3}{11}$ (b) $\frac{11}{6}$ (c) $\frac{11}{16}$ (d) $\frac{15}{7}$

4. $x = 0.5\overline{7}$ Express 'x' in fractional form the requires fraction will be
   (a) $\frac{26}{44}$ (b) $\frac{27}{45}$ (c) $\frac{26}{45}$ (d) $\frac{57}{100}$

5. $0.2\overline{45}$ in the simplest form will be equal to :
   (a) $\frac{49}{20}$ (b) $\frac{27}{110}$ (c) $\frac{22}{10}$ (d) $\frac{243}{9900}$

6. Which of the following number is rational?
   (a) π (b) $\frac{22}{7}$
   (c) $\sqrt{7}+2$ (d) 0.141141114...

7. If $\frac{\sqrt{3}+1}{2-\sqrt{3}} = x + y\sqrt{3}$, then x, y have values equal to
   (a) 3,5 (b) 5,3 (c) 3,4 (d) 3,6

8. $\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right) \times 50$ equals
   (a) 1000 (b) 200
   (c) 500 (d) 1500

9. If $x = 2+\sqrt{3}$, then $x + \frac{1}{x}$ is equal to :
   (a) $2\sqrt{3}$ (b) 4
   (c) 14 (d) 7

10. The value of the expression $\frac{16\times 2^{n+1} - 4\times 2^{n}}{16\times 2^{n+2} - 2\times 2^{n+2}}$ is
    (a) 2 (b) 2^n (c) $\frac{1}{2}$ (d) 4

11. Find the value of $x^3 - 2x^2 - 7x + 5$, if $x = \frac{1}{2-\sqrt{3}}$.
    (a) 1 (b) 0 (c) 2 (d) 3

12. If $\frac{\left(x^{a+b}\right)^2\left(x^{b+c}\right)^2\left(x^{c+a}\right)^2}{\left(x^a x^b x^c\right)^4} = y$ then y is equal to
    (a) x^{a+b+c} (b) 1 (c) x^{c+a} (d) 2

13. If $5^{x-3}\cdot 3^{2x-8} = 225$, $x = ?$
    (a) 4 (b) 3 (c) 5 (d) 6

14. $\sqrt{13-m\sqrt{10}} = \sqrt{8}+\sqrt{5}$, then $m =$
    (a) –2 (b) –5 (c) –6 (d) –4

15. $\left[2-3(2-3)^3\right]^3 = x$ then the value of $x = ?$
    (a) 125 (b) –125 (c) 25 (d) 625

16. $10^x = 64$, then the value of $10^{\frac{x}{2}+1}$ is
    (a) 8 (b) 6.4 (c) 640 (d) 80

17. $4^x - 4^{x-1} = 24$, then $(2x)^x$ is equal to
    (a) $\sqrt{5}$ (b) $125\sqrt{5}$ (c) $25\sqrt{5}$ (d) $5\sqrt{5}$

18. If $x^2 + \frac{1}{x^2} = 83$, then $x^3 + \frac{1}{x^3} =$
    (a) 756 (b)256 (c)729 (d) 702

19. If $x^2+\frac{1}{x^2}=98$, then $x+\frac{1}{x}=?$

(a) 10 (b) 12 (c) $7\sqrt{2}$ (d) 11

20. If $\frac{x}{y}+\frac{y}{x}=-1$, then $x^3-y^3=$

(a) –1 (b) $\frac{1}{2}$ (c) 1 (d) 0

21. If $x=7+4\sqrt{3}$ and $xy=1$ then $\frac{1}{x^2}+\frac{1}{y^2}=?$

(a) 64 (b)194 (c) $\frac{1}{49}$ (d) 134

22. If $x^{-2}=64$, then $x^0+x^{\frac{1}{3}}$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

23. $\left\{\left(23+2^2\right)^{\frac{2}{3}}+\left(140-19\right)^{\frac{1}{2}}\right\}^2$, is

(a) 324 (b) 400 (c) 196 (d) 289

24. The positive square root of $7+4\sqrt{3}$ is

(a) $7+\sqrt{3}$ (b) $7+2\sqrt{3}$

(c) $3+\sqrt{2}$ (d) $2+\sqrt{3}$

25. If $\sqrt{2}=1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(a) 2.4142 (b) 0.4142

(c) 5.8282 (d) 0.1718

26. If $\frac{3^{5x}}{3^{2x}}\times 81^2\times 6561=3^7$ then x

(a) 3 (b) $\frac{1}{3}$ (c) –3 (d) $-\frac{1}{3}$

27. $\frac{5^{n+2}-6\times 5^{n+1}}{13\times 5^n-2\times 5^{n+1}}$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$

28. If $x=1-\sqrt{2}$, then the value of $\left(x-\frac{1}{x}\right)^3$ is

(a) 4 (b) 27 (c)8 (d) –8

29. The value of $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{7}-\sqrt{6}}+\frac{1}{\sqrt{5}-2}$ is

(a) 5 (b) –5 (c) 4 (d) –4

30. If $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then $x+y+xy=$

(a) 5 (b) 7

(c)9 (d) 17

31. The square root of $5+2\sqrt{6}$ is

(a) $\sqrt{3},\sqrt{2}$ (b) $\sqrt{3},\sqrt{2}$

(c) $\sqrt{5},\sqrt{6}$ (d) $\sqrt{5},\sqrt{6}$

32. The value of 'm' for which $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{1}{4}}=7^m$ is

(a) –3 (b) 2 (c) $\frac{-1}{3}$ (d) $\frac{1}{4}$

33. If $2^{-m}\times\frac{1}{2^m}=\frac{1}{4}$ then $\frac{1}{14}\left\{\left(4^m\right)^{\frac{1}{2}}+\left(\frac{1}{5^m}\right)^{-1}\right\}$ is equal to

(a) 2 (b) $\frac{1}{2}$ (c)4 (d) $-\frac{1}{4}$

34. If $x=\sqrt{6}+\sqrt{5}$ then $x^2+\frac{1}{x^2}$

(a) $2\left(\sqrt{6}+1\right)$ (b) $2\sqrt{5}+2$

(c)20 (d) 22

35. $\frac{5-\sqrt{3}}{2+\sqrt{3}}=a+b\sqrt{3}$ then the respective values of a and b are

(a) 13, –7 (b) 13, 7
(c) –13, 7 (d) –13, –7

36. If $m - n = 1$ then $\dfrac{9^n \times 9 \times \left(3^{\frac{-n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \left(\dfrac{1}{3}\right)^x$ then $x =$

(a) 2 (b) –2 (c) 3 (d) –3

37. If $t = 8^2$ then $K = t^{\frac{2}{3}} + 4t^{\frac{-1}{2}}$ then $K =$

(a) $\dfrac{33}{2}$ (b) 1 (c) $\dfrac{257}{16}$ (d) $\dfrac{31}{2}$

38. $\left(\dfrac{243}{32}\right)^{-0.8} = t$, then the value of '$t$' will be

(a) $\dfrac{4}{9}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{8}{27}$ (d) $\dfrac{16}{81}$

39. If $\sqrt{5} = k$, then $\dfrac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ has value equal to

(a) $k\left(\sqrt{2}+1\right)$ (b) $k\left(\sqrt{2}-1\right)$

(c) $k\left(\sqrt{2}+3\right)$ (d) $k\left(2+\sqrt{2}\right)$

40. If $x = \dfrac{\sqrt{3}+1}{2}$, then the value of $4x^3 + 2x^2 - 8x + 7$ is

(a) 0 (b) 10 (c) 5 (d) 15

41. If $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$ then the value of $\dfrac{6}{\sqrt{5}-\sqrt{3}}$ is

(a) 11.904 (b) 10.904
(c) 3.968 (d) 8.968

Answer Key

1. (b)	2. (b)	3. (c)	4. (c)	5. (b)	6. (b)	7. (b)	8. (c)	9. (b)	10 (c)
11. (d)	12. (b)	13. (c)	14. (d)	15. (a)	16. (d)	17. (c)	18. (a)	19. (a)	20. (d)
21. (b)	22.(b)	23. (b)	24. (d)	25. (b)	26. (c)	27. (d)	28. (c)	29. (a)	30. (c)
31. (b)	32. (c)	33. (b)	34. (d)	35. (a)	36. (c)	37. (a)	38. (d)	39. (a)	40. (b)
41. (a)									

Hints *and* Solutions

1. **(b)** Zero is a whole number which is not a natural number. Every integer is a rational number. Every whole number is a integer but converse is false.

2. **(b)** Since,

— = 0.875 (Terminating decimal)

$$\sqrt{\frac{9}{125}} = \frac{3}{5\sqrt{5}} \text{ (Irrational)}$$

$$\frac{93}{300} = \frac{31}{100} = 0.31 \text{ (Terminating decimal)}$$

$$\frac{190}{30} = 6.\bar{3} \text{ (Repeating decimal)}$$

$\because$ Repeating and terminating decimals are rational numbers.

3. **(c)** $\because$ All the fractions are in their simplest form.

$\therefore$ The fraction having the denominator in the form $2^m \times 5^n$ will be terminating.

$\therefore$ Just analysing the denominators, we have 11,6 and 7 cannot be expressed in $2^m \times 5^n$ form , but $16 = 2^4 \times 5^0$.

$\therefore \frac{11}{16}$ will be a terminating decimal

4. **(c)** Given $x = 0.5\bar{7} = 0.5777$ ...(i)

then $10x = 5.777$ ...(ii)

and $100x = 57.777$ ...(iii)

Subtracting equation (ii) from equation (iii), we have

$$90x = 52$$

$$\Rightarrow \quad x = \frac{26}{45}$$

5. **(b)** Given $n = 0.2\overline{45}$ then $x = 0.24545$

$10x = 2.4545$ ...(i)

and $100x = 24.54545$

and $1000x = 245.454545$ ...(iii)

Subtracting eq (i) and eq (iii), we get

$$990x = 243$$

$$\Rightarrow \quad x = \frac{243}{990} = \frac{27}{110}$$

6. **(b)** $\pi = 3.14157......$

(Non-repeating non-terminating decimal)

$$\frac{22}{7} = 3.142871$$

$\therefore \frac{22}{7}$ is a rational number.

7. **(b)** Rationalising the denominator we have

$$\frac{\sqrt{3}+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{\left(\sqrt{3}+1\right)\left(\sqrt{3}+2\right)}{(2)^2-\left(\sqrt{3}\right)^2}$$

Let $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= 3 + 2 + 3\sqrt{3} = 5 + 3\sqrt{3} = x + y\sqrt{3}$$

$\therefore \quad x = 5, y = 3$

8. **(c)** Here

$$x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{3+2+2\sqrt{6}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} + \frac{3+2-2\sqrt{6}}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= 5 + 2\sqrt{6} + 5 - 2\sqrt{6} = 10$$

$\therefore$ Required value $= 10 \times 50 = 500$

9. **(b)** Given $x = 2+\sqrt{3}$ then

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$\therefore \quad x + \frac{1}{x} = \left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right) = 4$$

10. **(c)** We have $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$

$$= \frac{(2)^4 \times 2^{n+1} - (2)^2 \times 2^n}{(2)^4 \times (2)^{n+2} - 2 \times 2^{n+2}}$$

$$= \frac{2^{n+2}\left\{(2)^3-1\right\}}{2^{n+3}\left\{(2)^3-1\right\}} = \frac{1}{2}$$

11. **(d)** We have $x = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$

$$\Rightarrow \quad x - 2 = \sqrt{3}$$

Squaring both sides

$$(x-2)^2 = 3$$

$$\Rightarrow \quad x^2 + 4 - 4x = 3$$

$$\Rightarrow \quad x^2 - 4x + 1 = 0 \quad \text{...(i)}$$

$$x^3 - 2x^2 - 7x + 5$$

$$= x(x^2 - 4x + 1) + 2(x^2 - 4x + 1) + 3$$

$$= x \times 0 + 2 \times 0 + 3 = 0 + 3 = 3 \text{ (using eq (i))}$$

12. **(b)** Here $y = \frac{(x^{a+b})^2 \, (x^{b+c})^2 \, (x^{c+a})^2}{(x^a x^b x^c)^4}$

$$= \frac{(x^{a+b+b+c+c+a})^2}{(x^{a+b+c})^4}$$

$$= \frac{x^{4(a+b+c)}}{x^{4(a+b+c)}} = 1$$

13. **(c)** Given $5^{x-3} . 3^{2x-8} = 225 = 5^2 . 3^2$

$\therefore \quad x - 3 = 2x - 8 = 2$

$\Rightarrow \quad x = 5$

14. **(d)** Here $\sqrt{13 - m\sqrt{10}} = \sqrt{8} + \sqrt{5}$

Squaring both sides, we have

$13 - m\sqrt{10} = \left(\sqrt{8} + \sqrt{5}\right)^2$

$\Rightarrow \quad 13 - m\sqrt{10} = 8 + 5 + 2\sqrt{40}$

$\Rightarrow \quad -m\sqrt{10} = 2 \times \sqrt{4 \times 10}$

$\Rightarrow \quad -m\sqrt{10} = 2 \times 2\sqrt{10}$

$\Rightarrow \quad m = -4$

15. **(a)** Here $\left[2 - 3(2-3)^3\right]^3 = \left[2 - 3(-1)^3\right]^3$

$= [2+3]^3 = (5)^3 = 125 = x$

16. **(d)** $\because \; 10^x = 64 \Rightarrow \left(10^x\right)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8$

$\therefore \; 10^{\frac{x}{2}} = 8 \Rightarrow 10^{\frac{x}{2}+1} = 10^{\frac{x}{2}}.10 = 8.10 = 80$

17. **(c)** We have $4^x - 4^{x-1} = 24$

$\Rightarrow \quad 4^{x-1}(4 - 1) = 24$

$\Rightarrow \quad 4^{x-1} = 8$

$\Rightarrow \quad \frac{4^x}{4} = 8$

$\Rightarrow \quad 4^x = 32$

$\Rightarrow \quad (2)^{2x} = (2)^5$

$\Rightarrow \quad x = \frac{5}{2}$

$\therefore \quad (2x)^x = \left(2 \times \frac{5}{2}\right)^{\frac{5}{2}} = (5)^{\frac{5}{2}}$

$= (5)^{\frac{4}{2}} \cdot (5)^{\frac{1}{2}} = 25\sqrt{5}$

18. **(a)** $x^2 + \frac{1}{x^2} = 83$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x} - 2 = (83 - 2) = 81$

$\Rightarrow \quad \left(x - \frac{1}{x}\right) = 9$

$\therefore \quad \left(x - \frac{1}{x}\right)^3 = (9)^3$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3.x.\frac{1}{x}\left(x - \frac{1}{x}\right) = 729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3(9) = 729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 729 + 27 = 756$

19. **(a)** Here $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$= (98 + 2) = 100$

$\Rightarrow \quad x + \frac{1}{x} = \sqrt{100} = 10$

20. **(d)** We have $\frac{x}{y} + \frac{y}{x} = -1$

$\Rightarrow \quad x^2 + y^2 = -xy$

$\Rightarrow \quad x^2 + y^2 + xy = 0 \qquad$ ...(i)

We know that,

$x^3 - y^3 = (x - y)\left(x^2 + y^2 + xy\right) = (x - y)(0)$

[Using (i)]

$= 0$

21. **(b)** Here $xy = 1$

$$y = \frac{1}{x} = \frac{1}{7 + 4\sqrt{3}} = \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}}$$

$$= \frac{7 - 4\sqrt{3}}{(7)^2 - \left(4\sqrt{3}\right)^2} = \frac{7 - 4\sqrt{3}}{1} = 7 - 4\sqrt{3}$$

$\therefore \quad \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{x^2} + x^2$

$$= \left(x+\frac{1}{x}\right)^2 - 2 = (7+4\sqrt{3}+7-4\sqrt{3})^2 - 2$$

$$= (14)^2 - 2 = 196 - 2 = 194$$

22. **(b)** $\because \quad x^{-2} = 64$

$$\Rightarrow \quad x^{-1} = 8$$

$$\Rightarrow \quad x = \frac{1}{8}$$

$$\Rightarrow \quad x^{\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$$

$$x^0 + x^{\frac{1}{3}} = 1 + \frac{1}{2} = \frac{3}{2}$$

23. **(b)** The given equation can be written as

$$\left\{(23+4)^{\frac{2}{3}} + (121)^{\frac{1}{2}}\right\}^2$$

$$= \left\{(27)^{\frac{2}{3}} + (121)^{\frac{1}{2}}\right\}^2$$

$$= \left\{(3)^2 \aleph 11\right\}^2 \quad \{9 \quad 11\}^2$$

$$= [20]^2 = 400$$

24. **(d)** Let $7+4\sqrt{3} = \left(a+b\sqrt{3}\right)^2$

$$\Rightarrow \quad 7+4\sqrt{3} = a^2 + 3b^2 + 2ab\left(\sqrt{3}\right)$$

$$\Rightarrow \quad \left(a^2+3b^2\right) = 7, \; ab = 2$$

$\therefore\ a = 2,\ b = 1.$

$$\Rightarrow \quad \sqrt{7+4\sqrt{3}} = 2+\sqrt{3}$$

25. **(b)** Here $\dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} \times \dfrac{\sqrt{2}-1}{\sqrt{2}-1}$

$$= \frac{\left(\sqrt{2}-1\right)^2}{\left(\sqrt{2}\right)^2 - (1)^2}$$

$$= \frac{2+1-2\sqrt{2}}{2-1} = 3 - 2\sqrt{2}$$

Let, $\sqrt{3-2\sqrt{2}} = a + b\sqrt{2}$

$$\Rightarrow \quad 3-2\sqrt{2} = a^2 + b^2 \cdot 2 + 2ab\sqrt{2}$$

$$\Rightarrow a^2 + 2b^2 = 3, \; ab = -1$$

Solving these two equations, we have

$$a = -1, \; b = +1$$

$\therefore$ The required value $= \sqrt{2} - 1 = 1.4142 - 1$

$$= 0.4142$$

26. **(c)** $(3)^{5x-2x} \times (81)^2 \times 6561 = 3^7$

$$\Rightarrow \quad (3)^{3x} \times (3)^8 \times 81 \times 81 = 3^7$$

$$\Rightarrow \quad (3)^{3x} \times (3)^8 \times (3)^8 = 3^7$$

$$\Rightarrow \quad (3)^{3x+8+8} = 3^7$$

$$\Rightarrow \quad 3x + 16 = 7$$

$$\Rightarrow \quad x = \frac{7-16}{3} = \frac{-9}{3} = -3$$

27. **(d)** Here $\dfrac{5^{n+1}(5-6)}{5^n(13-10)} = \dfrac{5^{n+1}(-1)}{5^n(3)} = \dfrac{-5}{3}$

28. **(c)** $\dfrac{1}{x} = \dfrac{1}{1-\sqrt{2}} \times \dfrac{1+\sqrt{2}}{1+\sqrt{2}}$

$$= \frac{1+\sqrt{2}}{(1)^2 - \left(\sqrt{2}\right)^2} = \frac{1+\sqrt{2}}{1-2} = -\left(1+\sqrt{2}\right)$$

$$\Rightarrow \quad x - \frac{1}{x} = (1-\sqrt{2}) + 1 + \sqrt{2} = 2$$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^3 = (2)^3 = 8$$

29. **(a)** Here $\dfrac{1}{3-\sqrt{8}} = \dfrac{1}{3-\sqrt{8}} \times \dfrac{3+\sqrt{8}}{3+\sqrt{8}}$

$$= \frac{3+\sqrt{8}}{(3)^2 - (\sqrt{8})^2} = 3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}$$

$$= \sqrt{8}+\sqrt{7}$$

Similarly

$$\frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6}+\sqrt{5}, \; \frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{5}-2} = \sqrt{5}+2$$

Rearranging all the terms in the required pattern, we have

$$\left(3+\sqrt{8}\right)-\left(\sqrt{8}+\sqrt{7}\right)-\left(\sqrt{6}+\sqrt{5}\right)$$
$$+\left(\sqrt{7}+\sqrt{6}\right)+\left(\sqrt{5}+2\right)$$

$$= 3+\left(\sqrt{8}-\sqrt{8}\right)+\left(-\sqrt{7}+\sqrt{7}\right)$$
$$+\left(-\sqrt{6}+\sqrt{6}\right)+\left(-\sqrt{5}+\sqrt{5}\right)+2$$

$$= 3+2 = 5$$

30. **(c)** Given $x = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} = \frac{8+2\sqrt{15}}{2}$$

$$y = \frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} = \frac{8-2\sqrt{15}}{2}$$

Now $x+y+xy$

$$= \frac{8+2\sqrt{15}}{2}+\frac{8-2\sqrt{15}}{2}$$
$$+\left(\frac{8+2\sqrt{15}}{2}\right)\left(\frac{8+2\sqrt{15}}{2}\right)$$

$$= \frac{16}{2}+\frac{64-60}{4}$$
$$= 8+1 = 9$$

$$= 8+\frac{\{5-3\}^2}{2\times 2} = 8+\frac{4}{4} = 8+1 = 9$$

31. **(b)** Let $\sqrt{5+2\sqrt{6}} = \sqrt{a^2+b^2+2ab}$

$\Rightarrow \quad a^2+b^2 = 5,\ ab = \sqrt{6}$

Solving these two equations, we get

$$a = \sqrt{\ }\ ,\ b = \sqrt{2}$$

32. **(c)** $\left[\left\{\left(\dfrac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{1}{4}} = 7^m$

$\Rightarrow \quad \left\{(7)^{-2}\right\}^{-2\times\frac{-1}{3}\times\frac{1}{4}} = 7^m$

$\Rightarrow \quad (7)^{-2\times-2\times\frac{-1}{3}\times\frac{1}{4}} = 7^{\frac{-1}{3}} = 7^m$

$\Rightarrow \quad m = \dfrac{-1}{3}$

33. **(b)** $2^{-m}\times 2^{-m} = 2^{-2}$

$\Rightarrow \quad (2)^{-2m} = (2)^{-2}$

$\Rightarrow \quad m = 1$

Substituting the value of m in,

$$\frac{1}{14}\left\{\left(4^m\right)^{\frac{1}{2}}+\left(\frac{1}{5^m}\right)^{-1}\right\}$$

$$= \frac{1}{14}\left\{\left(4^{\frac{1}{2}}+\left(\frac{1}{5}\right)^{-1}\right)\right\}$$

$$= \frac{1}{14}\{2+5\} = \frac{1}{2}$$

34. **(d)** Given $x = \sqrt{6}+\sqrt{5}$

$\therefore \quad \dfrac{1}{x} = \dfrac{1}{\sqrt{6}+\sqrt{5}}\times\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

$$= \frac{\sqrt{6}-\sqrt{5}}{1} = \sqrt{6}-\sqrt{5}$$

Now $x^2+\dfrac{1}{x^2} = \left(x+\dfrac{1}{x}\right)^2-2$

$$= \left(\sqrt{6}+\sqrt{5}+\sqrt{6}-\sqrt{5}\right)^2-2$$

$$= \left(2\sqrt{6}\right)^2-2 = 24-2 = 22$$

35 (*a*) Here $\dfrac{5-\sqrt{3}}{2+\sqrt{3}} = \dfrac{5-\sqrt{3}}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{\left(5-\sqrt{3}\right)\left(2-\sqrt{3}\right)}{(2)^2-\left(\sqrt{3}\right)^2}$$

$= 10 + 3 - 7\sqrt{3} = a + b$

$= 13 - 7\sqrt{3} = a + b\sqrt{3}$

$\Rightarrow a = 13, b = -7$

36. **(c)** $\dfrac{9^n \times 9 \times 3^n - (27)^n}{3^{3m} \times 2^3} = \left(\dfrac{1}{3}\right)^x$

$\Rightarrow \dfrac{(27)^n [9-1]}{3^{3m} \times 8} = \left(\dfrac{1}{3}\right)^x$

$\Rightarrow \dfrac{(27)^n \times 8}{(27)^m \times 8} = \left(\dfrac{1}{3}\right)^x$

$\Rightarrow (3)^{3(n-m)} = (3)^{-x}$

$\Rightarrow x = 3(m - n) = 3 \qquad [m - n = 1]$

37. **(a)** Given $t = 8^2 = 64$

$\therefore \quad K = 8^{\frac{4}{3}} + 4(64)^{\frac{-1}{2}}$

$= (2)^4 + 4 \times \dfrac{1}{8}$

$= 16 + \dfrac{1}{2} = \dfrac{33}{2}$

38. **(d)** Here $\left(\dfrac{243}{32}\right)^{-0.8} = \left(\dfrac{32}{243}\right)^{0.8} = \left(\left(\dfrac{2}{3}\right)^5\right)^{0.8}$

$= \left(\dfrac{2}{3}\right)^4 = \dfrac{16}{81}$

39. **(a)** We have $\dfrac{5 \times 3}{\sqrt{5}\left(\sqrt{2} + \sqrt{4} + \sqrt{8} - 1 - \sqrt{16}\right)}$

$= \dfrac{\sqrt{5} \times \sqrt{5} \times 3}{\sqrt{5}\left(\sqrt{2} + 2 + 2\sqrt{2} - 1 - 4\right)}$

$= \dfrac{3\sqrt{5}}{\left(3\sqrt{2} - 3\right)} = \dfrac{\sqrt{5}}{\sqrt{2} - 1} \times \dfrac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= \sqrt{5}\left(\sqrt{2} + 1\right)$

$= k\left(\sqrt{2} + 1\right)$

40. **(b)** $\because \quad x = \dfrac{\sqrt{3} + 1}{2}$

$\therefore \quad x^2 = \dfrac{3 + 1 + 2\sqrt{3}}{4} = \dfrac{4 + 2\sqrt{3}}{4} = \dfrac{2 + \sqrt{3}}{2}$

and $x^3 = \dfrac{\left(\sqrt{3} + 1\right)^3}{8} = \dfrac{1 + 3\sqrt{3} + 3\sqrt{3}\left(\sqrt{3} + 1\right)}{8}$

$= \dfrac{10 + 6\sqrt{3}}{8} = \dfrac{5 + 3\sqrt{3}}{4}$

$\therefore \quad 4x^3 + 2x^2 - 8x + 7$

$= \left(5 + 3\sqrt{3}\right) + \left(2 + \sqrt{3}\right) - 8\left(\dfrac{\sqrt{3} + 1}{2}\right) + 7$

$= 14 + 4\sqrt{3} - 4\sqrt{3} - 4 = 10$

41. **(a)** We have $\dfrac{6}{\sqrt{5} - \sqrt{3}} = \dfrac{6}{\sqrt{5} - \sqrt{3}} \times \dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

$= \dfrac{\left(\sqrt{5} + \sqrt{3}\right)6}{2}$

$= 3\left(\sqrt{5} + \sqrt{3}\right)$

$= 3(2.236+1.732)$

$= 3(3.968)$

$= 11.904$

2 Polynomials

Learning Objective:

In this chapter we shall learn about :

- Polynomials and their types
- Factors of polynomials

Algebraic Expression

Expression separated by + or – operation are called the terms of algebraic expression.

Example: $9x + x^2 + x^3 + 4x^4$ is an algebraic expression and $9x$, x^2 , x^3 and $4x^4$ are the terms of the algebraic expression.

Coefficients

In the polynomial $7x^3 - 6x^2 + 8x + 4$, we say that coefficients of x^3, x^2 and x are 7, – 6 and 8 respectively and 4 is the constant term in it.

Polynomials

An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

Example: (i) $5x^3 - 5x^2 + 6x - 3$ is a polynomial in one variable x.

(ii) $x^2y + y^2z + 6x^3 + 7y^3$ is a polynomial in 3 variables, i.e, x,y and z.

Important Terms

Constants

A symbol having a fixed numerical value is called a constant.

Example: 2, 3, π, $\frac{-2}{3}$, – 6 etc. are constants.

Variables

A symbol which may be assigned different numerical values is known as a variable.

Example: In $C = 2\pi r$, C and r are variables.

Degree of a polynomial in one variable or more than one variable

In case of one variable, the highest power of the variable is called the degree of the polynomial

Example: (i) $2x + 5$ is a polynomial in x of degree 1.

(ii) $x^2 + 2x + 6$ is a polynomial in x of degree 2

In case of more than one variable, the sum of the powers of variables is taken into account, the highest sum so obtained is treated as the degree of the polynomial.

Example: (i) $7x^3 - 5x^2y^2 + 3xy + 6y + 8$ is a polynomial in y and x of degree 4.

Types of Polynomial

Zero Polynomial

The constant polynomial 0 is called the zero polynomial.

Linear polynomial

A polynomial of degree one is called a linear polynomial.

Quadratic polynomial

A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial

A polynomial of degree three is called a cubic polynomial.

Number of Terms in a Polynomial

(i) Monomial: A polynomial containing one nonzero term is called a monomial.

(ii) Binomial: A polynomial containing two non zero terms is called a binomial.
Example: $x - 5y$, $5x^2 + 2zx$

(iii) Trinomial: A polynomial containing three non- zero terms is called a trinomial.
Example: $x^3 + 5x^2 + 3y$, $x^2 + 3x + 9$, $xy + yz + x^2$ etc.

Constant Polynomial and Zero Polynomial

A polynomial containing one term only, i.e, constant term only is called a constant polynomial becomes equal to zero, the polynomial is said to be a zero polynomial.

Example 1: Which of the following expressions are polynomials ?

(a) $x^2 - 5x + 3$ (b) $2\sqrt{x} + 5$ (c) -8 (d) $3x^{\frac{2}{3}} + 6.$

Solution: (a) $\because$ The expression $x^2 - 5x + 3$ has all the non- negative integral powers in x.
$\therefore$ expression is a polynomial.

(b) $2\sqrt{x} + 5 = 2x^{\frac{1}{2}} + 5$

$\because$ x has a non-integral powers in n

$\therefore$ Given expression is not a polynomial.

(c) -8 is a constant term.

$\therefore$ This is a constant polynomial.

(d) $3x^{\frac{2}{3}} + 6$ has non– integral powers in x

$\therefore$ This is a not a polynomial.

Example 2: $x^2 + 5x - 2$ is polynomial of how many degrees and comment about number of terms in it?

Solution: $x^2 + 5x - 2$ is a polynomial in x of degree 2 and it has 3 terms.
$\therefore$ This polynomial is a binomial.

Example 3: $3x^3 + 3x^2 + 8x + 9$ is a polynomial in x. Classify this polynomial on the basis of degree and number of terms.

Solution: The highest power of x in the expression is 3.

$\therefore$ This polynomial $2x^3 + 3x^2 + 8x + 9$ is a cubic polynomial.

$\therefore$ Number of terms in polynomial = 4.

$\therefore$ This is conceded to be a 4 terms containing polynomial, i.e., quadronomial.

Factors of a Polynomial

Let $p(x)$ is a polynomial. If $p(a) = 0$ then 'a' is said to be a zero and $(x - a)$ is said to be a factor of polynomial $p(x)$.

Example 4: If $P(x) = x^2 + 3x + 4$, find $P(-2)$, $P(1)$.

Solution: $P(-2) = (-2)^2 + 3(x-2) + 4 = 4 - 6 + 4 = 2$

$$P(1) = (1)^2 + 3 \times 1 + 4$$
$$= 1 + 3 + 4 = 8.$$

Example 5: Find a zero of the polynomials

(a) $2x + 9$ (b) $4x - 8$

Solution: **(a)** Let $P(x) = 2x + 9$

Now $P(x) = 0$

$$\Rightarrow 2x + 9 = 0 \quad \Rightarrow \quad x = \frac{-9}{2}$$

(b) Let $P(x) = 4x - 8$

$$\text{Now, } P(x) = 0 \quad \Rightarrow \quad 4x - 8 = 0 \quad \Rightarrow \quad x = \frac{8}{4} = 2$$

Example 6: Find the coefficient of $5x^2 + 3x + 9$ in the expression $15x^4 + 9x^3 + 27x^2$.

Solution: Coefficient of $5x^2 + 3x + 9$ in the expression $15x^4 + 9x^3 + 27x^2 = 3x^2\,(5x^2 + 3x + 9)$, is $3x^2$.

Factorization

Factorization is a process of representing the given polynomial as a product of its factors which are of lower degree than the given polynomial.

Example: $x^2 - 4 = (x + 2)\,(x + 2)$

Important Formulae

(a) $(x+y)^2 = x^2 + y^2 + 2xy$

(b) $(x-y)^2 = x^2 + y^2 - 2xy$

(c) $x^2 - y^2 = (x+y)(x-y)$

(d) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(e) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

(f) $x^3 + y^3 = (x+y)(x^2 + y^2 - xy)$

(g) $x^3 - y^3 = (x-y)(x^2 + y^2 + xy)$

(h) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

(i) $(x-y+z)^2 = x^2 + y^2 + z^2 + 2(-xy - yz + zx)$

Example 7: Factorize: $x(x-y)^3 + 3x^2y(x-y)$.

Solution: We have $x(x-y)^3 + 3x^2y(x-y)$

$$= x\,(x-y)\,\{(x-y)^2 + 3xy\}$$
$$= x\,(x-y)\,\{x^2 + y^2 - 2xy + 3xy\}$$
$$= x\,[(x-y)\,(x^2 + y^2 + xy)]$$
$$= x\,(x^3 - y^3)$$

Example 8: Factorize :

(i) $x^2 + 3x + 3 + x$ (ii) $a^2 + b - ab - a$

Solution: (i) $x^2 + 3x + 3 + x$

$$= x\,(x+3) + 1(x+3) = (x+3)(x+1)$$

(ii) $a^2 + b - ab - a = a^2 - ab + b - a$

$$= a\,(a-b) - 1(a-b)$$
$$= (a-1)\,(a-b)$$

Example 9: Factorize:

(i) $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$ (ii) $x^2 + \frac{1}{x^2} - 2 - 3x - \frac{3}{x}$

Solution: (i) $\therefore \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$$\therefore\ x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x} = \left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right)$$
$$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$$

(ii) $\therefore \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$$\therefore\ x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x} = \left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right)$$
$$= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x} - 3\right)$$

Example 10: Factorize:

(i) $x^2 - (a+b)\,x + ab$ (ii) $x^3 - x^2 + ax + x - a - 1$

(iii) $(2x-3)^2 - 8x + 12$

Solution: (i) $x^2 - ax - bx + ab$

$$= x\,(x-a) - b\,(x-a)$$
$$= (x-a)\,(x-b)$$

(ii) $x^2\,(x-1) + x\,(a+1) - 1(a+1)$

$$= x^2\,(x-1) + (a+1)\,(x-1)$$
$$= (x-1)\,(x^2 + a + 1)$$

(iii) $(2x-3)^2-8x+12$

$$= (2x-3)(2x-3)-4(2x-3)$$
$$= (2x-3)(2x-3-4)$$
$$= (2x-3)(2x-7)$$

Example 11: Factorize :

(i) $a^2+2ab+b^2-4c^2$ (ii) x^2-y^2+6y-9

(iii) x^4-625 (iv) $3x^3-48x$

(v) $(a+b)^3-a-b$ (v) $(a+b)^3-a-b$

(vi) $9-a^2+2ab-b^2$

Solution: (i) $(a+b)^2-4c^2 = (a+b)^2-(2c)^2$

$$= (a+b+2c)(a+b-2c)$$

(ii) $x^2-(y^2-6y+9) = x^2-(y+3)^2$

$$= (x-y-3)(x+y+3)$$

(iii) $x^4-625 = (x^2)-(25)^2$

$$= (x^2-25)(x^2+25)$$
$$= (x+5)(x-5)(x^2+25)$$

(iv) $3x(x^2-16) = 3x(x+4)(x-4)$

(v) $(a+b)^3-a-b = (a+b)\{(a+b)^2-1\}$

$$= (a+b)(a+b+1)(a+b-1)$$

(vi) $9-a^2+2ab-b^2 = (3)^2-(a^2-2ab+b^2)$

$$= (3)^2-(a-b)^2$$
$$= (3+a-b)(3-a+b)$$

Example 12: Factorize :

(i) $2x^2-\frac{5}{6}x+\frac{1}{12}$ (ii) $\sqrt{3}x^2+11x+6\sqrt{3}$

Solution: (i) $\frac{2\times 12x \quad -10x+1}{12} = \frac{24x \quad -10x \quad 1}{12}$

$$= \frac{24x^2-6x-4x+1}{12}$$

$$= \frac{1}{12}(4x-1)(6x-1)$$

(ii) ax^2+bx+c, can be factorized as, multiply a and c, and express b as a sum of two numbers whose multiplication (product is equal to 'ac'

$\therefore \quad \sqrt{3}\times 6\sqrt{3} = 18$

$11x = 9x+2x$, also $9\times 2 = 18$

$$\therefore \quad \sqrt{3}x^2 + 11x + 6\sqrt{3} = \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3}$$

$$= \sqrt{3}x\left(x + 3\sqrt{3}\right) + 2\left(x + 3\sqrt{3}\right)$$

$$= \left(\sqrt{3}x + 2\right)\left(x + 3\sqrt{3}\right)$$

Example 13: Factorize :

(i) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}y^2 - 8xz$ (ii) Evaluate : $(97)^2$.

Solution: (i) $\left(-\sqrt{2}x\right)^2 + \left(y\right)^2 + \left(2\sqrt{2}z\right)^2 + 2\left(-\sqrt{2}xy + 2\sqrt{2}yz - 4xz\right) = \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

(ii) $(97)^2 = (100 - 3)^2 = (100)^2 + (3)^2 - 2 \times 100 \times 3$

$= 10000 + 9 - 600 = 9409$

Example 14: Expand $(3x + 2)^3$, and factorize $x^3 + 125$.

Solution: $(3x + 2)^3 = 27x^3 + 8 + 3 \times 3x \times 2(3x + 2)$

$= 27x^3 + 8 + 18x\ (3x + 2) = 27x^3 + 8 + 54x^2 + 36x$

$x^3 + 125 = (x)^3 + (5)^3 = (x + 5)\ (x^2 + 25 - 5x\ )$

Example 15: Evaluate : $x^3 + y^3 + z^3 - 3xyz$

Solution: $x^3 + y^3 + z^3 - 3xyz = \left(x\right)^3 + \left(y\right)^3 + z^3 - 3xyz$

$$= \left[\left(x^3\right) + \left(y\right)^3 + 3xy(x + y)\right] + z\left(z^2 - 3xy\right)$$

Let $x + y = u$

$= [4^2 - 3xy4] + z\ (z^2 - 3xy) = 4^3 + z^3 - 3xy\ (4 + z)$

$= (4 + z)\ [4^2 + z^2 - 4z - 3xy] = (4 + z)\ (x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)\ (x^2 + y^2 + z^2 - xy - yz - zx\ )$

Note

(i) If $x + y + z = 0$, then

$x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow \quad x^3 + y^3 + z^3 = 3xyz$

(ii) $x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2}\left[\left(x - y\right)^2 + \left(y - z\right)^2 + \left(z - x\right)^2\right]$

This can only be zero when, $x = y = z$

$\therefore$ when $x = y = z$, then also

$x^3 + y^3 + z^3 = 3xyz$

Example 16: Factorize :

(i) $\left(p - q\right)^3 + \left(q - r\right)^3 + \left(r - p\right)^3$

(ii) If $p + a = 2$ then what is the value of $a^3 + 6ap + p^3$?