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Prasoon Kumar

MATHS OLYMPIAD

International Mathematics Olympiad

8

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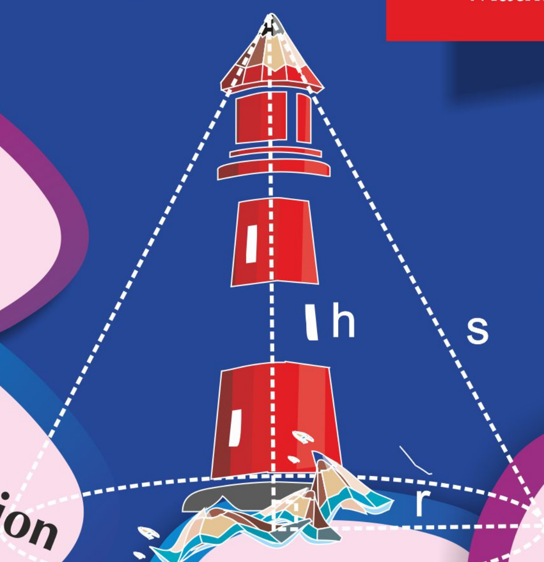

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INTERNATIONAL MATHEMATICS OLYMPIAD

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Publisher's Note

V&S Publishers, after the grand success of a number of academic and general books, is pleased to bring out a series of *Mathematics Olympiad books* under *The Gen X series – generating Xcellence in generation X* – which has been designed to focus on the problems faced by students. In all books the concepts have been explained clearly through various examples, illustrations and diagrams wherever required. Each book has been developed to meet specific needs of students who aspire to get distinctions in the field of mathematics and want to become Olympiad champs at national and international levels.

To go through Maths Olympiad successfully, students need to do thorough study of topics covered in the *Olympiads syllabus and the topics covered in school syllabus as well*. The Olympiads not only tests the subjective knowledge but Reasoning skills also. So students are required to comprehend the depth of concepts and problems and gain experience through practice. The Olympiads check efficiency of candidates in problem solving. These exams are conducted in different stages at regional, national, and international levels. At each stage of the test, the candidate should be fully prepared to go through the exam. Therefore, this exam requires careful attention towards comprehension of concepts, thorough practice, and application of rules and concepts.

While other books in market focus selectively on questions or theory; V&S Maths Olympiad books are rather comprehensive. Each book has been divided into five sections namely *Mathematics, Logical Reasoning, Achiever's section, Subjective section, and Model Papers*. The theory has been explained through solved examples. To enhance problem solving skills of candidates, *Multiple Choice Questions (MCQs)* with detailed solutions are given at the end of each chapter. Two *Mock Test Papers* have been included to understand the pattern of exam. A CD containing Study Chart for systematic preparation, Tips & Tricks to crack Maths Olympiad, Pattern of exam, and links of Previous Years Papers is accompanied with this book. The books are also useful for various competitive exams such as NTSE, NSTSE, and SLSTSE as well.

We wish you all success in the examination and a very bright future in the field of mathematics.

All the best

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Part A : Verbal Reasoning

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Section 1

Mathematical Reasoning

1 Playing with Numbers

Numbers in General Form

In general, any two digit number pq made of digits p and q can be written as,

$$pq = 10 \times p + q = 10p + q$$
$$qp = 10 \times q + p = 10q + p$$

Similarly,

For a 3-digit number pqr made of digits p, q and r can be written as,

$$pqr = 1000\ p + 100q + r$$

Example 1: Ravi took a number pq. He reversed the digits of the number and added the original number and obtained 187. If $p > q$, then find the values of p and q.

Solution: $pq = 10p + q$

After reversing the digits, $qp = 10q + p$

$\therefore$ $$pq + qp = (10p + q) + (10q + p)$$
$$= 11p + 11q = 11(p + q)$$

$\Rightarrow$ $$11(p + q) = 187$$

$\Rightarrow$ $$p + q = 17$$ [17 can be obtained by adding 9 and 8 only].

$\because p > q$, $\therefore p = 9$, $q = 8$.

Example 2: Sangam takes a number 'ab' and reversed its digits. Afterwards, he subtracted the new number from the original number to obtain 27 as a result. Find a and b, if, $ab > ba$.

Solution: $ab = 10a + b$

After reversing the digits, $ba = 10b + a$

$\therefore$ $$ab - ba = (10a + b) - (10b + a)$$
$$ab - ba = 9(a - b) = 72$$

$\Rightarrow$ $$a - b = 8 \qquad \ldots(i)$$

$\therefore$ $a = 9$, $b = 8$ only satisfies the above equation (i).

Example 3: If we take a number 'abc' and interchange the first and last digits to obtains 'cba', then,

$$abc - cba = 198$$
$$abc + cba = 88p$$

are obtained. Find p, a, b and c.

Solution: $abc = 100\ a + 10\ b + c$

$cba = 100\ c + 10\ b + a$

$\therefore$ $$abc - cba = 99\ (a - c) = 198$$

$\Rightarrow$ $$(a - c) = 2 \qquad \ldots(i)$$

$$abc + cba = 88p = 101(a+c) + 20b = 800 + 80 + p$$

$\Rightarrow \qquad 20b = 80$

$\Rightarrow \qquad b = 4,\ a + c = \dfrac{800 + p}{101}$ ...(ii)

p, a, c are natural numbers. Therefore, $p = 8, a + c = \dfrac{808}{101} = 8$ ...(iii)

$a = 5, c = 3, b = 4$ [From (i), (ii), (iii)].

$\therefore\ a = 5, b = 4, c = 3, p = 8.$

Letters For Digits

Here, some mathematical puzzles are solved using general techniques.

Example 4: Determine a, b, c, if,

$$\begin{array}{r} 4\,a \\ +\,9\,8 \\ \hline c\,b\,3 \\ \hline \end{array}$$

Solution: $4a = 40 + a$

$98 = 90 + 8$

$\therefore\ 4a + 98 = 138 + a$

$a + 8$ gives 3 on addition, if, $a = 5$

On addition 1, is left as a carry which is added with $(4 + 9)$.

$\therefore \qquad cb = 4 + 9 + 1 = 14$

$\therefore \qquad c = 1, b = 4, a = 5$

Example 5: $\begin{array}{r} 7\,a \\ \times\,6 \\ \hline a\,a\,a \\ \hline \end{array}$, Find a.

Solution: $7a = 70 + a.$

$\therefore \qquad (70 + a) \times 6 = 420 + 6a$

$\because\ a$ is a natural number, and, $(420 + 6a)$ give $(a\ a\ a)$ as a result.

$\therefore\ a = 4$

Example 6: Find a, b, c, if,

$$\begin{array}{r} a\,8\,3 \\ \times\,c\,9 \\ \hline a\,0\,4\,a \\ +\,1\,5\,b\,b\,0 \\ \hline c\,c\,a\,0\,a \\ \hline \end{array}$$

Solution: $3 \times 9 = 27$

$\therefore\ \ a = 7$

$$\therefore \quad \begin{array}{r} 7\,8\,3 \\ \times\, c\,9 \\ \hline 7\,0\,4\,7 \\ +1\,5\,b\,b\,0 \\ \hline c\,c\,7\,0\,7 \\ \hline \end{array}$$

$\because$ $4 + b$ gives such a number whose unit's place digit is zero.

$\therefore$ $b = 6$. Now, $\begin{array}{r} 7\,0\,4\,7 \\ +1\,5\,6\,6\,0 \\ \hline 2\,2\,1\,0\,7 \\ \hline \end{array}$

$\therefore$ $c = 2$

Tests of Divisibility

1. **Divisibility by 10:** Numbers whose units place is zero are divisible by 10, otherwise, not.
2. **Divisibility by 2:** If the one's digit of a number is 0,2,4,6,8 then the number is divisible by 2.
3. **Divisibility by 3 and 9:**
   (i) A number N is divisible by 9, if, the sum of its digits is divisible by 9.
   (ii) A number N is divisibility by 3, if, the sum of its digits is divisible by 3.
4. **Divisibility by 5:** If the one's digit of a number is 0 or 5, then the number is divisible by 5.
5. **Divisibility by 11:** A number *abcdefg* is divisible by 11, if,

$$(a + c + e + g) - (b + d + f) = 0,$$

Multiple Choice Questions

1. What is the next number in the series 5, 10, 26, 50, 122, ……
   (a) 129 (b) 170 (b) 204 (d) 138
2. A number is divisible by 63 if it is divisible by:
   (a) 7 (b) 9
   (c) both 7 and 9 (d) both 3 and 7
3. What is the sum of first 15 natural numbers?
   (a) 122 (b) 106 (c) 105 (d) 120
4. 7,835 + 2b1 = 8126, then the value of b will be :
   (a) 9 (b) 8 (c) 7 (d) 3
5. What is the sum of first 20 odd numbers?
   (a) 350 (b) 400 (c) 355 (d) 420
6. $\begin{array}{r} 1\,8\,2 \\ \times\, 2\,2 \\ \hline a\,0\,0\,a \\ \hline \end{array}$, the value of 'a' will be :
   (a) 2 (b) 3 (c) 4 (d) 8
7. What will be the last digit of 7^{333} ?
   (a) 1 (b) 7 (c) 3 (d) 9
8. Which of the following numbers is divisible by 11?
   (a) 1221 (b) 1223 (c) 1332 (d) 1343
9. $\begin{array}{r} 7\,3\,x\,5 \\ -\,2\,y\,7\,7 \\ \hline 4\,5\,1\,8 \\ \hline \end{array}$, the value of $(x + y)$ will be :
   (a) 16 (b) 17 (c) 18 (d) 15
10. $\begin{array}{r} a\,a \\ \times\, a\,a \\ \hline b\quad b \\ \hline \end{array}$, the value of (ab) will be : $(1 \le a, b \le 9)$
    (a) 32 (b) 16
    (c) 8 (d) 4
11. 763*312, which number should the * be replaced to make the number divisible by 9?
    (a) 7 (b) 5
    (c) 8 (d) 6

12. 76215*, the replacement of * by a number gives a number which is divisible by 11, the number will be :

(a) 8 (b) 7 (c) 6 (d) 9

13. $\begin{array}{r} A\,B\,C \\ A\,B\,C \\ +\,A\,B\,C \\ \hline B\,B\,B \\ \hline \end{array}$, the values of A, B, C are digits from 1 to 9. What will be value of B?

(a) 8 (b) 4 (c) 1 (d) 3

14. What will be the sum of first 22 natural numbers, which, are even?

(a) 506 (b) 406 (c) 484 (d) 253

15. One candle was guaranteed to burn for 6 hours, the other for 2 hours. They were both lit at same time. After some time one candle was twice as long as the other. For how long had they been burning?

(a) 3 hours (b) 6 hours

(c) $\frac{4}{3}$ hours (d) $\frac{3}{2}$ hours

16. Which is a 3-digit numbers, such that all its digits are prime and the 3 digits are the factors of the number?

(a) 735 (b) 537 (c) 435 9 (d) 245

17. Complete the square given below, and find the value of the sum of missing numbers. The sum of the magic square is 34.

5			
16		7	
	13		6
2		9	

t

(a) 68 (b) 39 (c) 78 (d) 84

18. Three numbers are such that their sum is 10 and their product is maximum. The product will be :

(a) 32 (b) 36 (c) 45 (d) 42

19. What will be the one's place digit of 6^{222}?

(a) 4 (b) 8

(c) 1 (d) 6

20. Find the smallest number which can be expressed as the sum of two cubes of natural numbers.

(a) 1729 (b) 1001 (c) 1728 (d) 1332

21. $\begin{array}{r} P\,A\,T \\ +\,E\,A\,T \\ \hline F\,E\,E\,A \\ \hline \end{array}$, where, P, A, T, E, F are digits from to what will be the value of F?

(a) 4 (b) 3 (c) 2 (d) 1

22. Sum of 3 numbers = product of 3 numbers. If the numbers are consecutive and natural. Find the triplet having least value, of their sum.

(a) 2, 3, 4 (b) 1, 2, 3

(c) 3, 4, 6 (d) 1, – 1, 0

23. The square of a number is having 5 at its units place and 2 at its tenths place, then the least natural number having these properties are :

(a) 5 (b) 15 (c) 25 (d) 4

24. The product 135 × 135 will be equal to :

(a) 19625 (b) 16925 (c) 18225 (d) 16235

25. Which of the following number is not a perfect squares ?

(a) 1024 (b) 441 (c) 1681 (d) 1282

26. $26 + 34 \times 17 \div 4 = 34$, which of the two signs should be interchanged to get the desired result?

(a) No change (b) ÷, +

(c) ×, ÷ (d) +, ×

27. Which is the least number divisible by 2, 3, 5 and 55 ?

(a) 110 (b) 550 (c) 660 (d) 330

28. What is the square number just greater than 60, which can be expressed as a sum of two successive triangular numbers?

(a) 72 (b) 64 (c) 81 (d) 100

29. What will be one's place digit for 9^{201}?

(a) 9 (b) 1 (c) 3 (d) 7

30. What is the value of P if P, Q, R are replaced by digits from 1 to 9? $PQ \times QP = RQPR$.

(a) 6 (b) 8 (c) 7 (d) 5

Answer Key

1. (b)	2. (c)	3. (d)	4. (a)	5. (b)	6. (c)	7. (b)	8. (d)	9. (b)	10. (c)
11. (b)	12. (b)	13. (b)	14. (a)	15. (c)	16. (a)	17. (c)	18. (b)	19. (d)	20. (a)
21. (d)	22. (b)	23. (a)	24. (c)	25. (d)	26. (c)	27. (d)	28. (b)	29. (a)	30. (b)

Hints and Solutions

1. **(b)** 5, 10, 26, 50, 122, …

$(2)^2+1, (3)^2+1, (5)^2+1,$

$(7)^2+1, (11)^2+1, (13)^2+1$

$\therefore$ Next number $= (13)^2 + 1 = 170$

2. **(c)** $63 = 7 \times 9$

$\therefore$ If a number is divisible by B, then it should be divisible by both 7 and 9.

3. **(d)** Sum of first 'n' natural numbers $= \dfrac{n(n+1)}{2}$

$\therefore$ Sum of first 15 natural numbers

$$= 15 \times \frac{(15+1)}{2}$$

$$= 15 \times 8 = 120$$

4. **(a)**

$$\begin{array}{r} \scriptstyle x \quad\;\; \\ 7\,8\,3\,5 \\ +\;2\,b\,1 \\ \hline 8\,1\,2\,6 \\ \hline \end{array}$$

Let 'x' be the carry, resulting from sum of b and 3.

$$8 + 2 + x = 11 \Rightarrow x = 1$$

$\therefore \quad 3 + b = 12$

$\Rightarrow \quad b = 9$

5. **(b)** Sum of 'n' odd natural numbers (first) $= n^2$

$\therefore$ Sum of first 20 odd natural numbers $= (20)^2$ $= 400$

6. **(c)** $182 \times 22 = 4004$

$\therefore \quad a = 4$

7. **(b)** $7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

$7^5 = 16807$

$\therefore$ After leaving 43 exponents 7^n repeats its unit digit.

$\therefore$ 7^{4n} has 1 as its units place digit.

$\therefore$ 7^{332} has 1 as units place digit.

$\therefore$ 7^{333} has 7 as its units place digit.

8. **(d)** If, sum of digits at odd place

= Sum of digits at even place.

Then , the number will be divisible by 11.

$\therefore$ 1343 is not divisible by 11.

9. **(b)**

$$\begin{array}{r} 4\,5\,1\,8 \\ +\;2\,y\,7\,7 \\ \hline 7\,3\,x\,5 \\ \hline \end{array}$$

When 8 and 7 are added, 1 is carry.

$\therefore \quad x = 7 + 1 + 1 = 9$

The carry of sum $(5 + y)$ should be 1.

$\therefore \quad 5 + y = 13$

$\Rightarrow \quad y = 8$

$\therefore \quad x + y = 17$

10. **(c)**

$$\begin{array}{r} a\;a \\ \times\;a\;a \\ \hline a^2\;a^2 \\ a^2\;a^2\;\times \\ \hline a^2\;\;2a^2a^2 \\ \hline \end{array} \qquad \begin{array}{r} a\;x \\ \times\;a\;a \\ \hline b\,8\,b \\ \hline \end{array}$$

$\Rightarrow \quad a^2 + a^2 = 8$

$\Rightarrow \quad 2a^2 = 8$

$\Rightarrow \quad a = 2, b = 4$

$\therefore \quad a \times b = 8$

11. **(b)** For a number to be divisible by 9, its digits sum should be divisible by 9.

$\therefore\ 7+6+3+*+3+1+2=22+*$.

$\therefore$ 5 should be written on place of * to make the number divisible by 9.

12. **(b)** 76215*

$\Rightarrow$ Sum of digits at odd places

$$= * + 1 + 6 = * = 7$$

Sum of digits at even places $= 5 + 2 + 7$

$$= 14$$

For divisibility with 11,

$$* + 7 = 14$$

$$\Rightarrow \quad * = 7$$

13. **(b)** $3c = x$ B, where, x is the carry,

$\therefore x + 3B = y\,B$, where y is the carry.

$\therefore y + 3A = B.$

$\therefore C = 8, B = 4, A = 1.$

$$\begin{array}{r} y\ x\ \\ A\ B\ C \\ A\ B\ C \\ +\ A\ B\ C \\ \hline B\ B\ C \\ \hline \end{array}$$

14. **(a)** Sum of first 22 even natural numbers

$$= (2 + 4 + \ldots\ldots + 44)$$

$$= 2(1 + 2 + \ldots\ldots + 22)$$

$$= 2 \times \frac{22 \times 23}{2}$$

$$= 2 \times 11 \times 23$$

$$= 22 \times 23$$

$$= 506$$

15. **(c)** Let the candles have burnt for 'x' hours,

$$\left(1 - \frac{x}{6}\right) = 2 \times \left(1 - \frac{x}{2}\right)$$

$$\Rightarrow \quad x = \frac{4}{3} \text{ hours.}$$

16. **(a)** 735 is a number having all its digits, a prime number and all the digits of 735 are the factors of 735.

17. **(c)**

5	x	e	d
16	y	7	c
a	13	b	6
2	2	9	f

We have to find, the value of

$$(a + b + c + d + e) + (x + y + z) + f = x$$

$\because$ Sum of numbers of any row/column = 34

$$\therefore\ a + b + 13 + 6 = x + y + z + 13$$

$$= e + b + 7 + 9 = c + d + 6 + f$$

$$= 5 + 16 + a + 2 = x + e + d + 5$$

$$= 16 + y + 7 + c = 2 + z + 9 + f = 34$$

$$\Rightarrow\ 2(a + b + c + d + e + f + x + y + z)$$

$$+ 19 + 13 + 16 + 6 + 23 + 5$$

$$+ 23 + 11 = 34 \times 8$$

$$\Rightarrow \quad 2x + 116 = 272$$

$$\Rightarrow \quad x = 78$$

18. **(b)** Let the natural numbers be, y and z.

$$\therefore \quad \frac{x + y + z}{3} \geq (xyz)^{\frac{1}{3}} \qquad [\text{AM} \geq \text{GM}]$$

$$\Rightarrow \quad \frac{10}{3} \geq (xyz)^{\frac{1}{3}}$$

$$\Rightarrow \quad xyz \leq \frac{1000}{27}$$

$$\Rightarrow \quad xyz \leq 37.03$$

$\therefore$ The limiting value of xyz is 37.03,

$\because$ The numbers x,y and z are natural.

$\therefore$ 37.03 cannot be obtained as a product.

$\because$ 37 is a prime number.

$\therefore$ 36 is the greatest number which can be obtained as a product of 3 natural numbers whose sum is 10.

19. **(d)** 60 = 1

$6^1 = 6$

$6^2 = 36$

$6^3 = 216$

$6^4 = 1296$

$6^5 = 7776$

$\therefore$ It is observed that 6^n has 6 at its units place.

$\therefore$ 6^{222} has 6 as its unit's place digit.

20. **(a)** $1729 = (12)^3 + (1)^3 = (10)^3 + (1)^3$

$\therefore$ 1729 can be expressed as sum of two perfect natural cubes.

It is the smallest number to satisfy this condition, and, is known as Ramanujan's Number.

21. **(d)**

$$\begin{array}{r} y\ x\ \\ P\ A\ T \\ +\ E\ A\ T \\ \hline F\ E\ E\ A \\ \hline \end{array}$$

x,y are respective carries.

$$y + P + E + F\,E$$
$$x + 2A = y\,E$$
$$T + T = x\,A$$

$\Rightarrow P = 9, A = 8, T = 4, E = 6$ and $F = 1$

$\therefore \quad F = 1$

22. **(b)** $xyz = x + y + z$ ...(i)

We also know that,

$$\left(\frac{x+y+z}{3}\right)^3 \geq xyz$$

$$\left(\frac{x+y+z}{3}\right)^3 - (x+y+z) \geq 0$$

$$(x+y+z)\left(\frac{(x+y+z)^2}{9} - 1\right) \geq 0$$

$$(x+y+z)\left(\frac{(x+y+z)^2 - 9}{2}\right) \geq 0$$

$$(x+y+z)\,(x+y+z-3)\,(x+y+z+3) \geq 0$$

$\therefore$ Sum of 3 natural numbers > 3.

$\therefore$ From general interpretation,

$$1 + 2 + 3 = 1 \Rightarrow 2 \Rightarrow 3$$

$\therefore$ Required set of natural numbers = (1, 2, 3).

23. **(a)** $\because$ Last (unit) place digit = 5

$\therefore$ The least natural number whose perfect square is having 5, as its unit place digit will be equal to 5.

$\therefore \quad 5^2 = 25$

24. **(c)** 135×135

Trick :

$135 \times 135 = 25$

Multiply the units place 5 and write the product, Add 1 to any one of the remaining digits at tenths place and write the product on tenths place.

$13 \times 14 = 182$

$\therefore$ $135 \times 135 = 18225$

25. **(d)** $\because$ Perfect squares should have 1, 4, 9, 6 and 5 as their units place digit.

$\therefore$ 1282 is not a perfect square.

26. **(c)** Interchange of $\times$, $\div$ will produce the desired result.

27. **(d)** Least number which is divisible by 2, 3, 5 and 55

= LCM 2, 3, 5 and 55

$= 66 \times 5$

= 330

28. **(b)** 60 < Sum of two triangular numbers = perfect square. The smallest number satisfying this condition is 64.

29. **(a)** 91 = 9

$9^2 = 81$

$9^3 = 243 \times 3 = 729$

$9^4 = 6561$

$9^5 = 59049$

$\therefore$ $9^{4n} = 6561$

$\therefore$ 9^{200} has 1 as its units place digit.

$\therefore$ 9^{201} has 9 as its units place digit.

30. **(b)**

$$\begin{array}{r} P\ Q \\ \times\ Q\ P \\ \hline R\ Q\ P\ R \\ \hline \end{array}$$

$P = 8, Q = 7, R = 6$

$\therefore \quad P = 8$

2 Rational Numbers

Rational Numbers

A number in the form of a/b, where, 'a' and 'b' are integers, and $b \neq 0$, is called a rational number.

Example: $\frac{-1}{2}, \frac{3}{5}, \frac{4}{90},$ etc.

Equivalent Rational Numbers

Two rational numbers $\frac{m}{n}$ and $\frac{p}{q}$ are equivalent, if $m \times q = n \times p$.

Infinite number of rational numbers, each of which is equivalent to a given rational number, can be written. The equivalent rational numbers can be obtained by multiplying the numerator and denominator of the given rational number by the same non-zero integer.

$\therefore$ $$\frac{a}{b} = \frac{a \times p}{b \times p} \text{ (where, } p \neq 0).$$

A rational number, whose numerator and denominator has only one common factor equal to 1 is said to be standard form of a rational number.

Terminating and Non-Terminating Decimals

If the denominator of a rational number, has, no other factors than 2, or, 5 or both is called terminating decimal, otherwise, the decimal will be non-terminating decimal.

Example: $\frac{1}{5}, \frac{1}{25}, \frac{1}{50}, \frac{1}{100}$ are terminating decimals and $\frac{1}{70}, \frac{70}{85}, \frac{14}{75}, \frac{17}{55}$ are non-terminating decimals.

First reduce the rational number into its standard form while, checking for terminating or non-terminating decimal.

Example 1: Which of the following rational numbers is the smallest ?

(a) $\frac{-15}{7}$, (b) $\frac{}{28}$

(c) $\frac{-25}{49}$, (d) $\frac{-35}{42}$.

Solution: LCM of denominators, *i.e.*, 7,28,49,42 is 588.

Now $$\frac{-15}{7} = \frac{-15 \times 84}{7 \times 48} = \frac{-1260}{588}$$

$$\frac{-5}{28} = \frac{-5 \times 21}{28 \times 21} = \frac{-105}{588}$$

$$\frac{-25}{49} = \frac{-25 \times 12}{49 \times 12} = \frac{-300}{588}$$

$$\frac{-35}{42} = \frac{-35 \times 14}{42 \times 14} = \frac{-490}{588}$$

$\because$ $\frac{-1260}{588}$ is the smallest of the four rational numbers.

$\therefore$ $\frac{-15}{7}$ is the smallest rational number among the given rational numbers.

Example 2: The sum of two rational numbers is $\frac{7}{8}$. If one of them is $\frac{1}{4}$, find the other number.

Solution: Let the other rational number be x.

$\therefore$ $$\frac{1}{4} + x = \frac{7}{8}$$

$\Rightarrow$ $$x = \frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$$

$\therefore$ Required rational number = 5/8.

Example 3: With what rational number should $\frac{-25}{\cdot}$ be multiplied to get $\frac{5}{14}$ as quotient (product)?

Solution: Let required number be x,

$\therefore$ $$\frac{-\ \cdot}{343} \times x = \frac{}{14}$$

$\Rightarrow$ $$x = \frac{5}{14} \times \frac{-343}{25} = \frac{-49}{10}$$

$\therefore$ Required number = $\frac{-49}{10}$

Example 4: Simplify :

$$\frac{7}{12} \times \frac{28}{3} - \frac{7}{10} \times \frac{5}{13} + \frac{2}{13} \times 169$$

Solution: We have $\frac{7}{12} \times \frac{28}{3} - \frac{7}{10} \times \frac{5}{13} + \frac{2}{13} \times 169$

$$= \frac{7}{12} \times \frac{28}{13} - \frac{7 \times 5}{10 \times 13} + 26$$

$$= \frac{7 \times 14}{6 \times 13} - \frac{7}{26} + 26 = \frac{7}{26}\left(\frac{14}{3} - 1\right) + 26$$

$$= \frac{7}{26} \times \frac{11}{3} + 26 = \frac{77}{78} + 26 = \frac{77 + 2028}{78} = \frac{2105}{78}.$$

Properties of Rational Numbers

1. **Commutative Property of Addition:** If x and y are two rational numbers, then $x + y = y + x$.
2. **Associative Property of Addition:** If x, y and z be any three rational numbers, then

$$(x + y) + z = x + (y + z).$$

3. **Property of Zero:** If x be any rational number, then,

$$0 + x = x + 0 = x, \text{ and,}$$
$$0 \times x = x \times 0 = 0.$$

4. **Additive/Multiplicative Inverse Property:** If x be any rational number, then, (– 0 is called the additive inverse of x, as.

$$x + (-x) = (-x) + x = 0.$$

If x $(x \neq 0)$ be any rational number, then, $\left(\frac{1}{x}\right)$ is called the multiplicative inverse of x, as.

$$x \times \frac{1}{x} = \frac{1}{x} \times x = 1.$$

5. **Associative Property of Multiplication:** If x, y and z be three rational numbers, then,

$$x \times (y \times z) = (x \times y) \times z.$$

7. **Distributive Property:** If x, y and z be any three rational numbers, then,

$$x \times (y + z) = xy + xz, \text{ or, } x \times (y - z) = xy - xz.$$

8. **Property of 1:** If x be any rational number, then $x \times 1 = 1 \times x = x$.

Example 5: If x, y, z be any three rational numbers, then which of the following alternatives is not true?

(a) $(x - y) + z = (x + y) + (y + z) - y$ (b) $x \div (y \div z) = (x \div y) \div (y \div z)$

(c) $(x + y) \times z = (x - y) + (y \times z)$ (d) $(x + y) \div z = (x \div z) + (y \div z)$

Solution: Considering option (b) $x \div (y \div z) = (x \div y) \div (y \div z)$

$\therefore$ Option (b) is not true.

Example 6: $x \times 0 = 0 \times x$, is true, but

$x \div 0 = 0 \div x$, is true or false? Justify.

Solution: $x \times 0 = 0 \times x = 0$ is satisfied by property of zero, but, if,

$x \div 0 = 0 \div x$, then, in L.H.S., 0 comes in denominator, which violates the condition of rational number.

Rational Numbers between Two Rational Numbers

- Between any two rational numbers, there exists infinitely rational numbers.
- If 'a' and 'b' be two rational numbers, such that,

$$a < b \text{ , then,}$$
$$a < \frac{a+b}{2} < b.$$

- Unlike natural numbers and integers, rational numbers do not have successors and predecessors.

Example 7: Find two rational numbers between $\frac{2}{13}$ and $\frac{5}{3}$.

Solution: LCM of 3 and 13 = 3 × 13 = 39.

$$\therefore \quad \frac{2}{13} = \frac{2 \times 3}{13 \times 3} = \frac{6}{39}$$

$$\frac{5}{13} = \frac{5 \times 13}{3 \times 13} = \frac{65}{39}$$

Between two integers 6 and 65, there are, 65 –6 –1 = 58 integers,

∴ We choose any two integers, say, 8, and 23, then,

$$\frac{6}{39} < \frac{8}{39} < \frac{23}{39} < \frac{65}{39}$$

∴ Two integers between $\frac{2}{13}$ and $\frac{5}{13}$ are $\frac{8}{39}$ and $\frac{23}{39}$.

Example 8: Find ten rational numbers between $\frac{2}{3}$ and 3.

Solution: LCM of 3 and 1 = 3 × 1 = 3.

$$\frac{2}{3} = \frac{2}{3}, \text{ and } \frac{3}{1} = \frac{3 \times 3}{1 \times 3} = \frac{9}{3}$$

Between two integers 2 and 9, there are, 9 – 2 –1 = 6 integers, but, we require 10 rational numbers.

∴ we multiply the common denominator by any natural number, which, is greater than 1, say. 3, then,

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6} \text{ and } \frac{3}{1} = \frac{9 \times 2}{3 \times 2} = \frac{18}{6}$$

∴ Ten integers between $\frac{2}{3}$ and 3 are

$$\frac{5}{6}, \frac{6}{6}, \frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \frac{11}{6}, \frac{12}{6}, \frac{13}{6}, \text{ or } \frac{5}{6}, 1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}, 2 \text{ and } \frac{13}{6}$$

Multiple Choice Questions

1. If $\frac{3}{5}$ of a number exceeds its $\frac{2}{7}$ by 44, then what is the number?

   (a) 144 (b) 148 (c) 140 (d)160

2. A bus is moving at an average speed of $60\frac{2}{5}$ km/hr. How much distance it will cover in $7\frac{1}{2}$ hour?

   (a) 423 km (b) 453 km
   (c) 443 km (d) 463 km.

3. The sum of two numbers is $\frac{-4}{3}$. If one of them is –5 then what is the other number?

   (a) $\frac{11}{3}$ (b) $\frac{-11}{3}$ (c) $\frac{16}{3}$ (d) $\frac{19}{3}$

4. In a school $\frac{5}{8}$ of the students are boys. If the number of girls are 270. What is the number of boys in the school?

(a) 440 (b) 450 (c) 420 (d) 400

5. A cord of length $58\frac{1}{2}$ m has been cut into 26 pieces of equal length. What is the length of each piece?

(a) $2\frac{1}{4}$ m (b) $\frac{32}{75}$ m (c) $\frac{64}{75}$ m (d) $\frac{8}{15}$ m

6. The product of two numbers is $\frac{-16}{75}$. If one of the number is $\frac{-15}{14}$ what is the other number?

(a) $\frac{16}{75}$ (b) $\frac{32}{75}$ (c) $\frac{64}{75}$ (d) $\frac{8}{15}$

7. What should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

(a) $\frac{-5}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{3}{2}$ (d) $\frac{-5}{4}$

8. What is additive inverse of $\frac{-7}{9}$?

(a) $\frac{7}{9}$ (b) $\frac{-9}{7}$ (c) $\frac{9}{7}$ (d) 1

9. The sum of two rational numbers is – 3. If one of the number is $\frac{-10}{3}$. What is the other number?

(a) $\frac{1}{3}$ (b) $\frac{13}{3}$ (c) $\frac{19}{3}$ (d) $\frac{-19}{3}$

10. The cost of $7\frac{1}{2}$ metres of cloth is ₹ $78\frac{3}{4}$. What is the cost of one metre of cloth?

(a) ₹ $13\frac{1}{2}$ (b) ₹ $10\frac{1}{2}$ (c) ₹ $16\frac{1}{2}$ (d) ₹ $12\frac{1}{2}$

11. By what number should $\frac{-33}{8}$ be divided to get $\frac{-11}{2}$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

12. By what rational number should we multiplying $\frac{-16}{63}$ to get $\frac{-4}{7}$.

(a) $\frac{7}{4}$ (b) $\frac{9}{4}$ (c) $\frac{3}{4}$ (d) $\frac{13}{4}$

13. What number should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?

(a) $\frac{75}{72}$ (b) $\frac{85}{72}$ (c) $\frac{83}{72}$ (d) $\frac{95}{72}$

14. What is reciprocal of $\left(\frac{1}{2}+\frac{1}{5}\right)$?

(a) $\frac{7}{10}$ (b) $\frac{10}{7}$ (c) $\frac{-7}{10}$ (d) $\frac{-10}{7}$

15. Which rational number is in between $\frac{-2}{3}$ and $\frac{-1}{4}$?

(a) $\frac{-5}{24}$ (b) $\frac{-5}{12}$

(c) $\frac{5}{12}$ (d) None of these

16. What is the reciprocal of $\left(\frac{1}{5}\times\frac{2}{5}\div\frac{4}{5}\right)$?

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) 10 (d) 5

17. What should be added to $\frac{-3}{5}$ to get $\frac{-1}{3}$?

(a) $\frac{4}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{15}$ (d) $\frac{8}{15}$

18. What is the additive inverse of $\left(\frac{3}{4}-\frac{2}{3}+\frac{1}{5}\right)$?

(a) $\frac{17}{60}$ (b) $\frac{-17}{60}$ (c) $\frac{60}{17}$ (d) $\frac{-60}{17}$