584 LECTURE NOTES IN ECONOMICS AND MATHEMATICAL SYSTEMS

Charlotte Bruun
Editor

Advances in Artificial Economics

The Economy as a Complex Dynamic System

Springer

Lecture Notes in Economics and Mathematical Systems

584

Charlotte Bruun (Ed.)

Advances in Artificial Economics

The Economy as a Complex Dynamic System

With 93 Figures
and 30 Tables

Springer

Editor

Professor Charlotte Bruun
Department of Economics, Politics
and Public Administration
Aalborg University
9100 Aalborg, Denmark
cbruun@socsci.aau.dk

ISBN-10 3-540-37247-4 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-37247-9 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media
springeronline.com

Typesetting: Camera ready by author
Cover: Erich Kirchner, Heidelberg
Production: LE-T_EX, Jelonek, Schmidt & Vöckler GbR, Leipzig

SPIN 11812852 Printed on acid-free paper – 88/3100 – 5 4 3 2 1 0

Preface

The symposium "Artificial Economics 2006" is the second in a planned line of symposia on artificial economics, following a symposium held in Lille, France in 2005, organized by Phillipe Mathieu, Bruno Beaufils and Olivier Brandouy [1]. The organizing theme of these symposia, is the computational study of economies perceived as complex dynamic systems.

With the latter being a non-existing phenomenon, the defining distinction is not between *artificial* and *natural* economics, but rather between aiming to understand economic processes by constructively simulating them, as opposed to reductionistically analyzing economic systems. With this distinction the game is set, and doors are open for new understandings of economic systems.

Artificial economics is a methodological approach rather than a paradigmatic approach. Neoclassicals, Keynesians, Marxists etc. may all benefit from the methods of artificial economics. Surely some New Classicals have felt the straight jacket of eg. having to assume homogeneous or representative agents, and certainly many Keynesians have dreamt of unifying microeconomics and macroeconomics without totally giving up on their macromodel. Artificial economics provide a toolbox fit for turning towards such fundamental problems anew, without adopting a predetermined idea of what the answers are going to be.

What artificial economics does embrace is an encouragement to economics and economic subdisciplines, to take off the blinkers, and learn about other disciplines. Artificial economics encompasses implementation of ideas and modeltypes from other sciences into economics, integration of different economic submodels, as well as the export of economic conceptions to other sciences. The three invited speakers of Artificial Economics 2006, Akira Nametame, Thomas Lux and Kumaraswamy "Vela" Velupillai, together with a number of contributors, all prove that much may be gained by moving between disciplines.

Akira Nametame, from the Department of Computer Science, National Defense Academy, Yokosuka, Japan, has moved between the fields of physics, computer science and economics - or more generally, social sciences. With applied physics and operations research as his original fields, Nametame has in recent years commuted between economics and computer science, managing to enrich both fields with his interdisciplinary insights. In his speak, printed as Chapter 11 in this volume, Nametame will discuss the formation of social norms by means of interaction (network effects).

Thomas Lux, Department of Economics, University of Kiel, Germany, started his career in macroeconomics, but has made important contributions to finance by introducing new tools adapted from other sciences to the field. Among other contributions, he was one of the first to apply statistical mechanics to financial markets [3]. Following up on this theme, Lux has combined behavioural finance, agent-based computational economics and econophysics in order to explain the stylized facts of financial returns (eg. fat tails and volatility). In his speak Thomas Lux will discuss estimation of agent-based models.

Kumaraswamy "Vela" Velupillai, National University of Ireland, Galway, Ireland and Trento University, Italy, moves elegantly between several economic subdisciplines with macroeconomics as his home base, and a well-founded knowledge of mathematics, computability theory, philosophy etc. He is the founder of "Computable Economics" [2], i.e. a discipline in which results and theoretical tools stemming from classical recursion theory are applied to study fundamental economic problems with special reference to the computability, constructivity and complexity of economic decisions, institutions and environments. K. Velupillai has proven himself as a strong methodological watch dog - watching over both the analytical and the artificial approaches to economics, and this is also the position he shall take in his speak.

The Artificial Economics conferences are two-day symposia - a form that served its purpose well in Lille 2005 by generating interesting discussions between subfields - discussions that would not have arisen, had each subfield gone to different parallel sessions. The drawback is the limited number of papers that this form leaves room for. Again this year, space only permitted half of the submitted extended abstracts to be accepted. The difficult selection process was based on a double-blind reviewing process, where each paper was send to three referees. A thanks to all submitters of extended abstracts - without you there could be no symposium.

The Scientific Committee of Artificial Economics 2006 did a great job in reviewing submitted papers and broadcasting news about the Symposium. Thank You!

Scientific Committee:

- Frédéric AMBLARD - Université de Toulouse 1, France
- Robert AXTELL - Brookings Institute, USA
- Gérard BALLOT - ERMES, Université de Paris 2, France
- Bruno BEAUFILS - LIFL, USTL, France
- Giulio BOTTAZZI - S.Anna School, Pisa, Italy
- Paul BOURGINE - CREA, École Polytechnique, France
- Olivier BRANDOUY - CLAREE, USTL, France
- Charlotte BRUUN, Aalborg University, Denmark
- José Maria CASTRO CALDAS - ISCTE, DINAMIA, Portugal
- Christophe DEISSENBERG - GREQAM, France
- Jean-Paul DELAHAYE - LIFL, USTL, France
- Wander JAGER - University of Groningen, The Netherlands
- Marco JANSSEN - Arizona State University, USA
- Philippe LAMARRE - LINA, Université de Nantes, France
- Luigi MARENGO St. Anna school of advanced studies, Pisa, Italy
- Philippe MATHIEU - LIFL, USTL, France
- Denis PHAN - Université de Rennes I, France
- Juliette ROUCHIER - GREQAM, France
- Sorin SOLOMON - The Hebrew University of Jerusalem, Israel
- Leigh TESFATSION - Iowa State University, USA
- Elpida TZAFESTAS - National Technical University of Athens, Greece
- Murat YIDILZOGLU - IFREDE-E3i, Université Montesquieu Bordeaux IV, France
- Stefano ZAMBELLI - Aalborg University, Denmark

With special thanks to Bruno Beaufils, Olivier Brandouy and Philippe Mathieu, for putting their “symposium template” in my hands and guiding me all the way through, and to Stefano Zambelli for that little extra help and encouragement that means so much.

Aalborg,
June 2006 *Charlotte Bruun*

References

[1] Amblard, F., O. Brandouy and P. Mathieu (eds)(2005), Artificial Economics - Agent-Based Methods in Finance, Game Theory and Their Application. Lecture Notes in Economics and Mathematical Systems 564. Springer Berlin Heidelberg New York

[2] Velupillai, K. (2000) Computable Economics. Oxford University Press

[3] Lux, T. (1995) Herd Behaviour, Bubbles and Crashes. Economic Journal 105 1995 pp. 881-896

Contents

Part I Market Structure and Economic Behaviour

Part II Market Efficiency and the Role of Speculation

Part III Firm-Consumer Dynamics

Part IV Social Interaction - Network Effects

Part V Social Interaction - Connectivity

Part VI Methodological Issues and Their Application

List of Contributors

Mikhail Anufriev
University of Amsterdam
The Netherlands

Alexandre T. Baraviera
UFRGS
Brasil

Ana L. C. Bazzan
UFRGS
Brasil

Andrew Bertie
Open University
UK

Olivier Brandoy
LEM, UMR CNRS-USTL
France

Andrea Consiglio
Università di Palermo
Italy

Silvio R. Dahmen
UFRGS
Brasil

S.A. Delre
University of Groningen
The Netherlands

Pietro Dindo
University of Amsterdam
The Netherlands

J.H. von Eije
University of Groningen
The Netherlands

Giorgio Fagiolo
University of Verona
Sant'Anna School of Advanced Studies
Italy

César García-Díaz
University of Groningen
The Netherlands

Alexander Gorobets
Sevastopol National Technical University
Ukraine

Cesáreo Hernández
University of Valladolid
Spain

Susan Himmelweit
Open University
UK

A.O.I. Hoffmann
University of Groningen
The Netherlands

Wander Jager
University of Groningen
The Netherlands

Toshiji Kawagoe
Future University
Japan

Valerio Lacagnina
Università di Palermo
Italy

Jacques Laye
LEF Inra Sae2/Engref
France

Maximilien Laye
Laboratoire d'Économétrie de l'École Polytechnique
France

Marco LiCalzi
University of Venice
Italy

Charis Lina
University of Crete
Greece

Adolfo López-Paredes
University of Valladolid
Spain

Craig Lynch
Macquarie University
Australia

Philippe Mathieu
LIFL, UMR CNRS-USTL
France

Peter McBurney
University of Liverpool
UK

Alessio Moneta
Max Planck Institute of Economics
Germany

Thierry Moyaux
University of Liverpool
UK

Akira Namatame
National Defense Academy
Japan

Bart Nooteboom
Tilburg University
The Netherlands

J. Pajares
University of Valladolid
Spain

Stéphane Pajot
University of Rennes 1
France

Valentyn Panchenko
University of Amsterdam
The Netherlands

José A. Pascual
University of Valladolid
Spain

Paolo Pellizzari
U. of Venice
Italy

Denis Phan
University of Rennes 1
France

Marta Posada
University of Valladolid
Spain

Srinivas Raghavendra
National University of Ireland, Galway
Ireland

Annalisa Russino
Università di Palermo
Italy

Shinichi Sasaki
Future University
Japan

Roberto da Silva
UFRGS
Brasil

Sitabhra Sinha
CIT Campus
India

Hervé Tanguy
Laboratoire d'Économétrie de l'École Polytechnique
France

Andrew Trigg
Open University
UK

Paul Windrum
Manchester Metropolitan University Business School
University of Maastricht
UK / The Netherlands

Arjen van Witteloostuijn
University of Durham
UK

Part I

Market Structure and Economic Behaviour

1

Heterogeneous Beliefs Under Different Market Architectures

Mikhail Anufriev[1] and Valentyn Panchenko[2]

[1] CeNDEF, University of Amsterdam, Amsterdam `m.anufriev@uva.nl`
[2] CeNDEF, University of Amsterdam, Amsterdam `v.panchenko@uva.nl`

Summary. The paper analyzes the dynamics in a model with heterogeneous agents trading in simple markets under different trading protocols. Starting with the analytically tractable model of [4], we build a simulation platform with the aim to investigate the impact of the trading rules on the agents' ecology and aggregate time series properties. The key *behavioral* feature of the model is the presence of a finite set of simple beliefs which agents choose each time step according to a fitness measure. The price is determined endogenously and our focus is on the role of the *structural* assumption about the market architecture. Analyzing dynamics under such different trading protocols as the Walrasian auction, the batch auction and the 'order-book' mechanism, we find that the resulting time series are similar to those originating from the noisy version of the model [4]. We distinguish the randomness caused by a finite number of agents and the randomness induced by an order-based mechanisms and analyze their impact on the model dynamics.

1.1 Introduction

The paper contributes to the analysis of the interplay between behavioral ecologies of markets with heterogeneous traders and institutional market settings. The investigation is motivated by the aim to explain inside a relatively simple and comprehensible model those numerous "stylized facts" that are left unexplained in the limits of the classical financial market paradigm (see e.g. [3]). Since the dynamics of financial market is an outcome of a complicated interrelation between behavioral patterns and underlying structure, it seems reasonable to start with an analytically tractable model based on realistic behavioral assumptions and to simulate it in a more realistic market setting. Such a strategy is chosen in this paper.

The first generation of agent-based models of financial markets followed the so-called bottom-up approach. The models were populated by an "ocean" of boundedly rational traders with adaptive behavior and were designed to be simulated on the computers. The Santa Fe artificial market (AM) model [1, 9] represents one of the best known examples of such approach. See also

[10] and reviews in [7] and [8]. The inherent difficulty to interpret the results of simulations in a systematic way led many researchers to build the models with heterogeneous agents which can be rigorously analyzed by the tools of the theory of dynamical systems. The achievements of the latter approach are summarized in [6]. In particular, the evolutionary model of Brock and Hommes (henceforth BH model) introduced in [4] follows the ideas of the Santa Fe AM in that the traders repeatedly choose among a finite number of predictors of the future price according to their past performance.

All the models mentioned so far (both simulational and analytic) are based on a simple framework with the mythical Walrasian auctioneer clearing the market. Real markets are functioning in a completely different way, and many recent models try to capture this fact. For instance, in [11] it is shown that an artificial market with a realistic architecture, namely an order-driven market under electronic book protocol, is capable of generating satisfactory statistical properties of price series (e.g. leptokurtosis of the returns distribution) in the presence of homogeneous agents. Similarly, the agent-based simulations in [2] demonstrate that the architecture bears a central influence on the statistical properties of returns. The latter contribution is also focused on the interrelation between market architecture and behavioral ecology, and in this respect is closely related to our paper. We relax, however, the assumption of a “frozen” population made in [2], and allow the agents to update their behavior over time.

More specifically, we assume that before the trading round, each agent can choose one of two simple predictors for the next price. The individual demand function depends on the predictor chosen, while the price is fixed later according to the specific market mechanism. The choice of predictor is implemented as a random draw with binary choice probabilities depending on the relative past performances of two predictors. An important parameter of the model is the intensity of choice, which measures the sensitivity of the choice probability to the relative performance. The higher the intensity of choice, the higher the probability that the best performing predictor is chosen. We simulate and compare the market populated by such heterogeneous agents under three aggregating mechanisms: Walrasian auction, batch auction, and an “order-book” mechanism. The latter two cases are interesting, since they resemble two protocols implemented in real stock exchanges. On the other hand, simulation of the Walrasian scenario provides a well-understood benchmark. Indeed, when the number of agents tends to infinity, our stochastic model converges to the deterministic BH model, thoroughly analyzed in [4].

In this paper, we show that understanding the basic mechanisms of the BH model can be very helpful also when dealing with more realistic market architecture. Indeed, the qualitative aspects of the non-linear dynamics generated by the BH model turn out to be surprisingly robust with respect to the choice of the market mechanism. Nevertheless, there are some important effects which realistic mechanisms supplement to the model. First, the finiteness of the number of agents provides a stabilizing effect on the model, since it implies a bigger noise in the choice of the predictor, which is equivalent to a smaller intensity of choice. Second, the inherent randomness of the markets under order-driven protocols (when agents have to choose one or few points from their demand curves) add destabilizing noise, which can be amplified, when the fundamental equilibrium is unstable. As a result, the generated time series remind the noisy version of the BH dynamics, when the system is switching between different attractors. This result is now produced, however, without adding either exogenous (e.g. due to the dividend realizations), or dynamic noise to the model. Third, we investigate the impact of two types of orders, market and limit orders, on the dynamics. We introduce a new parameter, the agents' propensity to submit market orders, which determines agent's preferences in submitting market orders as opposite to limit orders. We show that when this propensity high, the dynamics under the batch auction greatly deviate from the underlying fundamental, while the dynamics of the order-driven market converges to the dynamics under the Walrasian scenario. We also show some descriptive statistics for return time series generated for different values of the intensity of choice and the propensity to submit market orders.

The rest of the paper is organized as follows. In the next section we present the deterministic BH model, focusing on the agents' behavior, which is modeled in a similar way in our simulations. We also briefly discuss the properties of the dynamics for different values of the intensity of choice. In Section 1.3, we explain the three market mechanisms and introduce the difference between market and limit orders. Simulations results are presented and discussed in Section 16.5. Section 19.5 points to possible directions for future research.

1.2 The Brock-Hommes Benchmark Model

Let us consider a market where two assets are traded in discrete time. The riskless asset is perfectly elastically supplied at gross return $R = 1 + r_f$. At the beginning of each trading period t, the risky asset pays a random dividend y_t which is an independent identically distributed (i.i.d.) variable with mean $\bar{y}$. The price at period t is determined through a market-clearing condition (Walrasian scenario) and denoted by p_t. In the case of zero total supply of the risky asset, the fundamental price, which we denote by p_f, is given by the discounted sum of the expected future dividends $\bar{y}/r_f$. This is also the solution to the market-clearing equation for the case of homogeneous rational expectations.

In modeling the agents' behavior we closely follow the BH approach taken in [4]. Traders are mean-variance optimizers with absolute risk aversion a. Their demand for the risky asset reads

$$D_{i,t}(p_t) = \frac{E_{i,t-1}[p_{t+1} + y_{t+1}] - (1 + r_f)\, p_t}{a\, V_{i,t-1}[p_{t+1} + y_{t+1}]}\,, \tag{1.1}$$

where $E_{i,t-1}[p_{t+1} + y_{t+1}]$ and $V_{i,t-1}[p_{t+1} + y_{t+1}]$ denote the expectations of trader i about, respectively, the mean and variance of price cum dividend at time $t+1$ conditional upon the information available at the end of time $t-1$. It is assumed that all the agents expect the same conditional variance σ^2 at any moment t, and that there are different predictors for the mean. Thus, the agents in the model have heterogeneous expectations.

We concentrate here on one of a few cases analyzed in [4] and assume that two predictors are available in the market, *fundamental* and *trend-chasing.* These two predictors capture, in a very stylized way, two different attitudes observed in real markets. The *fundamental* predictor forecasts the fundamental value $p^f = \bar{y}/r_f$ for the next period price, so that

$$E_t^1[p_{t+1} + y_{t+1}] = p^f + \bar{y}\,.$$

According to the trend-chasing predictor, the deviations from the fundamental price p^f can be persistent, i.e.

$$E_t^2[p_{t+1} + y_{t+1}] = (1-g)\,p^f + g\,p_{t-1} + \bar{y}\,,$$

for some positive g.

In the BH model the population of agents is continually evolving. Namely, at the beginning of time t, agents choose one predictor among the two, according to their relative success, which in turn depends on the *performance measure* of predictors. The fraction n_t^h of the agents who use predictor $h \in \{1,2\}$ is determined on the basis of the average profit π_{t-1}^h obtained by the traders of type h between periods $t-2$ and $t-1$. Since under the Walrasian market-clearing, all agents with a given predictor have the same profit, the average profit of a type in the BH model can be simply referred as the profit of a given type.

As soon as the profit π_{t-1}^h is determined, the performance measure U_{t-1}^h of strategy h can be computed. Agents have to pay a positive cost C per time unit to get an access to the fundamental strategy, and $U_{t-1}^1 = \pi_{t-1}^1 - C$, while the trend-chasing strategy is available for free, and hence, $U_{t-1}^2 = \pi_{t-1}^2$. In our simulation model, we, in addition, apply a transformation to this performance measure to make it scale-free: $\tilde{U}_{t-1}^h = U_{t-1}^h/(|U_{t-1}^1| + |U_{t-1}^2|)$. Finally, the fraction n_t^h is given by the discrete choice model, so that

$$n_t^h = \exp[\beta \tilde{U}_{t-1}^h]/Z_{t-1}\,, \quad \text{where} \quad Z_{t-1} = \sum\nolimits_h \exp[\beta \tilde{U}_{t-1}^h]\,. \tag{1.2}$$

The key parameter β measures the *intensity of choice*, i.e. how accurately agents switch between different prediction types. If the intensity of choice is infinite, the traders always switch to the hystorically most successful strategy. On the opposite extreme, $\beta = 0$, agents are equally distributed between different types independent of the past performance.

Let us briefly discuss the dependence of the price dynamics on the intensity of choice in the BH model. For details the reader is refereed to [4], where

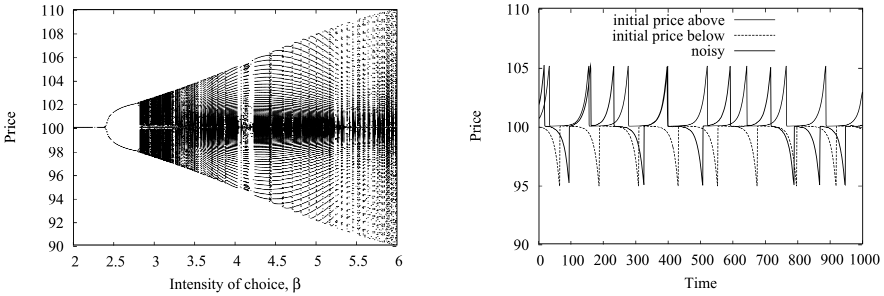

Fig. 1.1. Time-series properties of the Brock-Hommes model. **Left Panel:** Bifurcation diagram with respect to intensity of choice β. For each $\beta \in (2, 6)$, 500 points after 1000 transitory periods are shown for two different initial conditions: one below fundamental price and one above. The parameters are $C = 1$, $g = 1.2$, $r_f = 0.1$ and $\bar{y} = 10$. **Right Panel:** Typical time series for intensity of choice after the secondary bifurcation, in this case for $\beta = 4$. See text for explanation.

the deterministic skeleton with constant dividend is analyzed. From the bifurcation diagram shown in the left panel of Fig. 1.1, it can be seen that the fundamental equilibrium, where the price is equal to p_f, is stable for small values of β. For $\beta = \beta^* \approx 2.35$, a primary pitchfork bifurcation occurs, where the fundamental equilibrium loses stability. Two additional stable equilibria appear, one above and one below the fundamental and the original equilibrium becomes unstable. (Notice that for each β we show the prices for two initial conditions, belonging to the basins of attraction of two different equilibria.) A secondary Neimark-Sacker bifurcation takes place for $\beta = \beta^{**} \approx 2.78$. A stable quasiperiodic cycle emerges immediately afterward. With higher β the amplitude of this cycle increases, so that it almost touches the unstable fundamental equilibrium. For $\beta = \infty$ the system is close to a homoclinic bifurcation, which explains the typical time series for high β, reproduced in the right panel of Fig. 1.1.

If the initial price $p_0 > p_f$, then the price will grow (shown by solid thin line), further diverging from the unstable fundamental equilibrium. The trend following behavior, which is dominating due to its zero costs, is responsible for this market bubble. The forecasted error of trend-followers increases over time, however, since the actual price grows faster than expected. When the error becomes too high, it offsets the positive cost C of fundamental predictor. From this moment agents prefer to switch to fundamental behavior, contributing to a crash. From (1.2) it can be seen that, due to finite β, some small fraction of chartists remains in the market. This fact keeps the price a bit above the fundamental value and new bubble starts. A similar pattern with negative bubbles can be observed for initial price $p_0 < p_f$ (shown by the thin dotted line in the right panel of Fig. 1.1). Finally, if a small amount of dynamical noise is added, the positive and negative bubbles coexist on the trajectory (shown by the thick solid line). The observed behavior is qualitatively the same for all relatively high β, only the amplitude of the quasi-periodic cycle increases with β, as can be seen from the bifurcation diagram.

It is important to stress that the time series described above are obtained under the assumption of a constant dividend. Thus, the BH model is able to explain the excess volatility as an endogenous outcome of the agents' interactions. A more sophisticated model built in a similar spirit in [5] concentrates on the explanation of other stylized facts. The authors reproduce volatility clustering and realistic autocorrelation, kurtosis and skewness of the return distribution. Since the main goal of this paper is an investigation of the impacts of the market mechanisms on the model, but not the reproduction of the stylized facts, we will limit our analysis in the next sections to the simplest possible BH model.

1.3 Different Market Designs

On the basis of the analytic BH model, we construct an agent-based model and investigate its behavior under different trading protocols. In the agent-based model, the fraction n_t^h is interpreted as a probability of agent i to be of type h. The Walrasian auction is set as a benchmark, since the standard argument of the Law of Large Numbers implies that its outcome is equivalent to the original BH model as the number of agents tends to infinity. We will compare this setting with two more realistic order-driven markets, i.e. the batch auction and order book. Thought the paper we consider continuous prices.

1.3.1 Walrasian Auction

Under the Walrasian auction, at time t each agent i submits his excess demand function $\Delta D_{i,t}(p)$, which is the difference between his demand $D_{i,t}$ defined in (1.1) and his current position in the risky asset. The price p_t is determined from the market clearing condition $\sum_i \Delta D_{i,t}(p_t) = 0$. Notice that the equilibrium price p_t is always unique for the considered demand functions.

1.3.2 Batch Auction

Under the batch auction mechanism, each agent submits one or more orders, instead of the whole demand function. There are two types of the orders: limit and market order. A *limit order* consists of a price/quantity combination (p, q). Similarly to [2], an agent determines the price of a limit order as $p = p^* \pm \varepsilon|p_{t-1} - p^*|$, where p^* is the solution to the agent's "no-rebalancing condition" $\Delta D_{i,t}(p^*) = 0$, ε is a random variable, uniformly distributed on $[0, 1]$, and "+" corresponds to sell order and "−" to buy order. The quantity of the limit order at price p is given by $q = \Delta D_{i,t}(p)$. A *market order* specifies only the desired quantity of shares. As in [2], the type of order is determined by a propensity to submit a market order $m \in [0, 1]$, which is exogenously given parameter. A limit order (p, q) becomes a market order $(\cdot, q)$, if $\varepsilon < m$ in the limit order price equation. The price p_t is determined as an intersection of demand and supply schedules build on the basis of submitted orders (see [2] for details). Market buy/sell orders are priced at the min/max price among the corresponding side limit orders, which guaranties their fulfillment.

1.3.3 Order Book

In the order-book market, a period of time does not correspond to a single trade any longer. Instead, there is one trading session over period t and price p_t is the closing price of the session. Each agent can place only one buy or sell order during the session. The sequence in which agents place their orders is determined randomly.

During the session the market operates according to the following mechanism. There is an electronic book containing unsatisfied agents' buy and sell orders placed during current trading session. When a new buy or sell order arrives to the market, it is checked against the counter-side of the book. The order is partially or completely executed if it finds a *match*, i.e. a counter-side order at requested or better price, starting from the best available price. An unsatisfied order or its part is placed in the book. At the end of the session all unsatisfied orders are removed from the book.

As in the batch auction setting, there are two types of the orders: limit and market orders. The mechanisms for determining type of the order, its price and quantity are equivalent to those described in Section 1.3.2. The quantity of the market order is determined from the excess demand on the basis of the last transaction price.

1.4 Simulation Results

In Fig. 1.2, 1.3 and 1.4 we present the outcomes of typical simulations for different market architectures, different values of intensity of choice parameter β and different propensity to market orders. Ignoring transitory 1000 points, we show in each panel 4 time series, corresponding to the equilibrium price in the deterministic BH model (solid thick line), the equilibrium price in the simulated agent-based model under Walrasian (solid thin line) and batch (dashed line) auctions, and, finally, the closed price under order-book protocol (dotted line). Apart from the first two simulations, all the results are reported for 500 agents present in the market. For each β we compare the case $m = 0.1$, when nearly all orders are limit orders, with $m = 0.8$, when the majority of the orders are of market type. Finally, we consider five following values of intensity of choice. First, $\beta = 2.5$, which lies between two bifurcation values β^* and

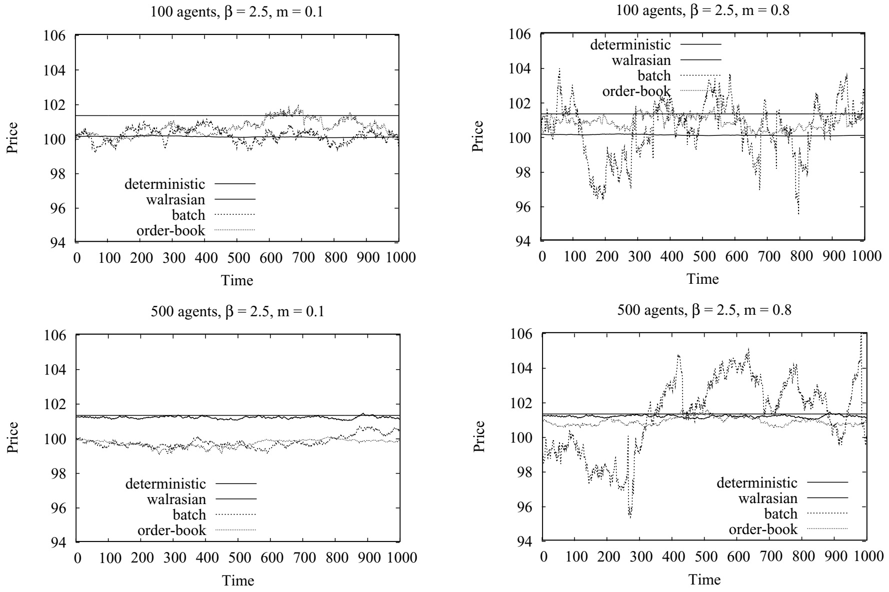

Fig. 1.2. Price time series under different market mechanisms (see the legends) for different number of agents and different propensity to submit market orders m (see the titles). Intensity of choice $\beta = 2.5$.

β^{**} (see Fig. 1.2). Then, $\beta = 2.75$ and $\beta = 2.8$, i.e. immediately before and after the secondary bifurcation (see Fig. 1.3). And finally, $\beta = 3$ and $\beta = 5$, i.e. far above β^{**}, when the quasi-periodic dynamics discussed at the end of Section 19.2 has already emerged (see Fig. 1.4).

For $\beta = 2.5$ the fundamental equilibrium is unstable, and the stable equilibrium of the BH model lies above $p_f = 100$, at the level $p^* \approx 101.3$. When the number of agents is small (as in the upper panels of Fig. 1.2), the discrepancy between the theoretical fraction of fundamentalists, n_t^f, computed according to (1.2) and the realized fraction is relatively large. Such discrepancy can be thought of as the agents' mistake in the computation of the performance measure. Therefore, it corresponds to a smaller "effective" intensity of choice with respect to $\beta = 2.5$. It explains why the relatively stable time series of Walrasian scenario lies well below the BH benchmark, close to $p_f = 100$: this is simply stable steady-state for some smaller value of β. When the number of agents increases, the error between the theoretical and realized fraction of fundamentalists decreases and the Walrasian scenario is getting closer to the BH benchmark (see the lower panels of Fig. 1.2).

The higher level of noise, which is intrinsic to the order-driven markets, has similar stabilizing consequences for the remaining two market mechanisms. This can be clearly seen in the lower left panel of Fig. 1.2, where price for both

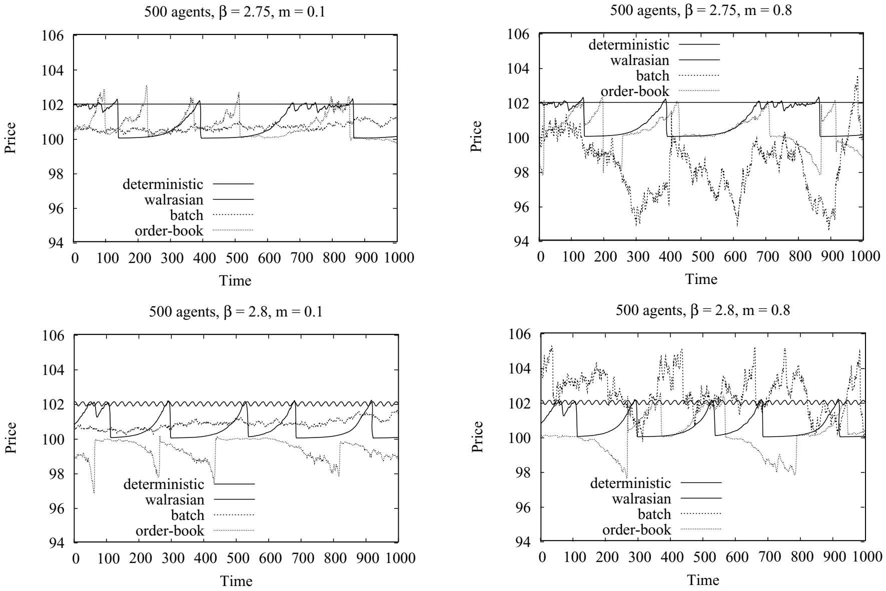

Fig. 1.3. Price time series under different market mechanisms (see the legends) for 500 agents, different intensities of choice and different propensity to submit market orders m (see the titles).

batch and order-book markets fluctuates around equilibrium, which is stable only for some smaller value of β. This stabilizing "β-effect" takes place also for other parametrizations, but usually cannot be seen, since other destabilizing effects dominate.

For example, in the right panels of Fig. 1.2, one can clearly see that an increase of the propensity to submit market orders m has strong destabilizing effect on the batch auction. It is interesting that the same increase of m has rather stabilizing consequences for the order-book mechanism and shifts the price towards the benchmark fundamental value (cf. 1.2, the two lower panels). This should not, however, come as a surprise, given the difference between these two mechanisms. Indeed, under the order-book, the executed prices of the market orders always come from some limit orders. Thus, the realized prices are still mainly determined by the limit orders, while increasing the randomness from the higher propensity to submit market orders m, probably leads to the stabilizing "β-effect" which we discussed above. On the other hand, under the batch protocol with many market orders, the price becomes very dependent on the relative sizes of buy and sell market orders and, therefore, its realization becomes more random by itself.

The two upper panels of Fig. 1.3 reveal another effect, implied by two types of randomness, i.e. one due to the errors between the theoretical and

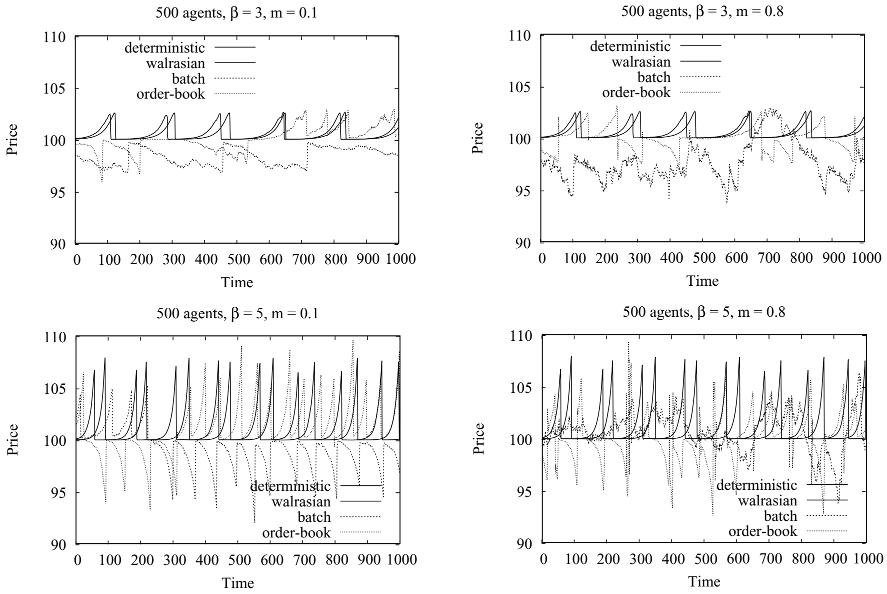

Fig. 1.4. Price time series under different market mechanisms (see the legends) for 500 agents, different intensities of choice and different propensity to submit market orders m (see the titles).

the realized fraction of traders, and one inherent in order-driven markets. Here, the BH model still generates stable dynamics converging to $p^* \approx 102$. The dynamics under the Walrasian auction and the order-book are unstable, however. The reason for this is a very small size of the basin of attractor p^*. The small endogenous noise constantly drives the dynamics out of this attractor, even if it ultimately comes back due to the instability of the fundamental fixed point. In addition, we again observe that "β-effect" has strong stabilizing effect for the batch auction with small propensity to submit market orders, $m = 0.1$. If the propensity is high, $m = 0.8$, the batch auction again leads to a very unstable behavior with large fluctuations and, sometimes, outliers. Similar characteristic can be given to the case $\beta = 2.8$, which is shown on the lower panels of Fig. 1.3.

Finally, Fig. 1.4 gives examples for relatively high values of β, when the stabilizing "β-effect" does not play a role, since the secondary bifurcation has already occurred under all market mechanisms. The main inference is that the analytical BH model based on the Walrasian auction is able to replicate the dynamics under more sophisticated trading mechanisms quite well. In particular, the time series in the two lower panels resemble the one obtained in the right panel of Fig. 1.1, when the dynamical noise triggers the dynamics between the two coexisting quasi-periodic attractors.

Table 1.1. Deceptive statistics of the return series generated under various market settings.

Auction	Walrasian	Batch	Order-Book	Walrasian	Batch	Order-Book
		$m = 0.1$			$m = 0.8$	
$\beta = 2.50$						
mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
variance	0.0002	0.0005	0.0003	0.0002	0.0033	0.0004
skewness	−0.178	−0.040	−0.468	−0.178	−7.760	−0.033
kurtosis	0.357	0.046	1.153	0.357	123.384	0.631
$\beta = 2.75$						
mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
variance	0.0007	0.0005	0.0012	0.0007	0.0027	0.0024
skewness	−12.693	−0.063	−0.410	−12.693	0.941	2.241
kurtosis	191.460	0.040	97.315	191.460	19.251	181.319
$\beta = 2.80$						
mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
variance	0.0009	0.0005	0.0017	0.0009	0.0026	0.0019
skewness	−12.766	0.129	−9.356	−12.766	−2.141	13.710
kurtosis	185.355	0.073	118.671	185.355	22.256	407.478
$\beta = 3.00$						
mean	0.0000	0.0000	0.0000	0.0000	0.0005	0.0000
variance	0.0014	0.0010	0.0021	0.0014	0.0269	0.0034
skewness	−13.151	−12.891	−10.871	−13.151	−10.757	−0.471
kurtosis	183.716	234.023	138.080	183.716	243.943	103.755
$\beta = 5.00$						
mean	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001
variance	0.0059	0.0054	0.0091	0.0059	0.0030	0.0133
skewness	−10.602	−7.965	−4.988	−10.602	0.199	1.265
kurtosis	115.248	66.308	47.613	115.248	13.820	43.380

Table 1.1 shows descriptive statistics of the return series for various β and m under different market auctions. In most cases the values of the skewness and kurtosis are far from realistic (e.g. S&P series returns statistics reported in [5]). Nevertheless, for $\beta = 5$ and $m = 0.8$ the values of the statistics for the batch and order-book auctions become closer to the realistic values.

1.5 Conclusion

The analytically tractable BH model introduced in [4] is quite successful in reproducing a number of stylized facts. Indeed, when the intensity of choice in this model is high, the price time series may deviate from fundamental benchmark in a systematic way, become quasi-periodic or even chaotic, and exhibit excess volatility. The phenomenon of volatility clustering can also be reproduced in a similar framework, as discussed e.g. in [5]. However, the unrealistic market clearing scenario, where each agent has to supplement an infinite amount of information to an (in)famous Walrasian auctioneer, has always cast a shadow on such an explanation of the stylized facts.

The results of this paper suggest that the order-based model is able to replicate the main features of the evolutionary BH model. Moreover, we found that the finiteness of the number of agents provides stabilizing effect, which is equivalent to a lower intensity of choice β in the deterministic model. The randomness resulting from the batch auction and the order-book mechanism destabilizes the model. This effect is mainly observed when the basins of attraction of the steady state (cycle) are small, i.e. in the vicinity of a bifurcation.

While investigating the effects of the limit- and market order, we found that the presence of the large number of market orders may substantially destabilize the dynamics of the batch auction. Instead, under the book-order mechanism, this effect is not observed.

The analysis of the descriptive statistics of the return series for different parameters and under different market protocols suggests that the structural assumptions are able to explain only some stylized facts, e.g. excess kurtosis. The model did not generate volatility clustering under any protocol, which suggest that this phenomenon should be modeled using the appropriate behavioral assumptions.

This result brings us to the directions for the future research. It would be interesting to start with a more realistic model (e.g. the model [5]), which is able to reproduce volatility clustering, and investigate its dynamics under various market mechanisms. Moreover, we could adopt different mechanisms for the limit order price generation, which are closer to those observed on the real markets. On the behavioral level, we could distinguished some parameters (e.g. β) between agents within one group and introduce a memory parameter into the individual type selection procedure.

References

[1] Arthur WB, Holland JH, LeBaron B, Palmer R, Tayler P (1997) Asset pricing under endogenous expectations in an artificial stock market. In: Arthur WB, Durlauf SN, Lane D (eds) The economy as an evolving complex system II: 15-44. Addison-Wesley

[2] Bottazzi G, Dosi G, Rebesco I (2005) Institutional architectures and behavioral ecologies in the dynamics of financial markets: a preliminary investigation. Journal of Mathematical Economics 41: 197-228

[3] Brock WA (1997) Asset price behavior in complex environments. In: Arthur WB, Durlauf SN, Lane DA (eds) The economy as an evolving complex system II: 385-423. Addison-Wesley

[4] Brock WA, Hommes CH (1998) Heterogeneous beliefs and routes to chaos in a simple asset pricing model. Journal of Economic Dynamics and Control 22: 1235-1274

[5] Gaunersdorfer A, Hommes CH (2005) A nonlinear structural model for volatility clustering. In: Teyssière G, Kirman A (eds) Long Memory in Economics: 265-288. Springer

[6] Hommes CH (2006) Heterogeneous agent models in economics and finance. In: Judd K, Tesfatsion L (eds) Handbook of Computational Economics II: Agent-Based Computational Economics. Elsevier, North-Holland

[7] LeBaron B (2000) Agent-based computational finance: suggested readings and early research. Journal of Economic Dynamics and Control 24: 679-702

[8] LeBaron B (2006) Agent-based computational finance. In: Judd K, Tesfatsion L (eds) Handbook of Computational Economics II: Agent-Based Computational Economics. Elsevier, North-Holland

[9] LeBaron B, Arthur WB, Palmer R (1999) Time series properties of an artificial stock market. Journal of Economic Dynamics and Control 23: 1487-1516

[10] Levy M, Levy H and Solomon S (2000) Microscopic simulation of financial markets. Academic Press, London

[11] LiCalzi M, Pellizzari P (2003) Fundamentalists clashing over the book: a study of order-driven stock markets. Quantitative Finance 3: 1-11

2

The Allocative Effectiveness of Market Protocols Under Intelligent Trading

Marco LiCalzi[1] and Paolo Pellizzari[2]

[1] Dept. Applied Mathematics and SSAV, U. of Venice, Italy `licalzi@unive.it`
[2] Dept. Applied Mathematics and SSAV, U. of Venice, Italy `paolop@unive.it`

2.1 Introduction

An important criterion for the evaluation of an exchange market is its ability to achieve allocative efficiency. The seminal paper by Gode and Sunder (1993) shows that the protocol known as continuous double auction can attain the efficient allocation even if the traders exhibit "zero-intelligence": hence, market protocols may actively contribute to the discovery of an efficient allocation. This paper spawned a variety of computer simulations that "enabled us to discover that allocative efficiency [...] is largely independent of variations in individual behavior" at least in canonical environments; see Sunder (2004).

However, the attainment of allocative efficiency is only a necessary condition for the effectiveness of a market protocol in an exchange economy. For instance, consider the fictitious protocol of Walrasian tâtonnement, where a centralized market maker iteratively elicit traders' excess demand functions and adjust prices before trade takes actually place. Under standard conditions, this protocol attains allocative efficiency while simultaneously minimizing both the volume of transactions and price dispersion. Moreover, the efficient allocation is reached in one giant step, so that its speed of convergence (after trade begins) is instantaneous.

Clearly, the Walrasian mechanism is only an idealization. Realistic market protocols require far less information from traders and should not be expected to perform as smoothly. This raises the question of ranking the effectiveness of those different market protocols which are commonly used in real markets; see Audet et al (2002) or Satterthwaite and Williams (2002). Assuming that they all pass the test of achieving an efficient allocation, which additional criteria should enter in their comparison? Walrasian tâtonnement suggests at least three possibilities: excess volume, time to convergence, and price dispersion.

A major complication in the study of alternative protocols is that their outcome is profoundly affected by traders' behavior; see Brewer et al (2002). This may exhibit sophisticated strategies, behavioral biases, access to different forecasting abilities, and a variety of factors which we encompass under the term of traders' *intelligence*. Gode and Sunder (1993) introduced the notion of "zero intelligence" as an extreme assumption, under which all complications in traders' behavior are ruled out and traders are only requested to satisfy a natural budget constraint. They argued that the outcome of a market protocol under zero intelligence is a test of its intrinsic ability to perform effectively.

Assuming zero intelligence, LiCalzi and Pellizzari (2005) compares the performance of different market protocols with regard to allocative efficiency and other criteria such as excess volume or price dispersion. The main protocols examined are: the batch auction, the continuous double auction, a (nondiscretionary) specialist dealership, and a hybrid of these last two. All the four protocols exhibit a remarkable ability to achieve allocative efficiency under three variants of zero intelligence, confirming the main insight from Gode and Sunder (1993).

However, even under zero intelligence, stark differences in performance emerge over other relevant dimensions. The continuous double auction has the worst performance with respect to excess volume, time to convergence, and price dispersion. The dealership has a lower time to convergence and never performs worse than the batch auction. These differences are sometimes dramatic and sometimes small (but persistent). Hence, LiCalzi and Pelllizzari (2005) concludes that (under zero intelligence) there is a clear partial ranking of these protocols with respect to excess volume, time to convergence, and price dispersion. A dealership performs slightly better than a batch auction or a hybrid market, and both are substantially more effective than a continuous double auction.

The relevance of this conclusion for the evaluation of practical market protocols is severely limited by the assumption of zero intelligence, which rules out the impact of differences in traders' behavior. The question addressed in this paper is how much of this conclusion remains true if we remove zero intelligence. Using two simple rules for intelligent trading, we study the performance ranking for the four market protocols with regard to excess volume, time to convergence, and price dispersion.

The organization of the paper is the following. Section 2.2 describes the model used in our simulations. Section 2.3 details the experimental design. Section 2.4 reports on the results obtained and Section 2.5 offers our conclusions. For an expanded and more robust analysis, see LiCalzi and Pellizzari (2006).

2.2 The Model

We use the same setup as in LiCalzi and Pellizzari (2005), where a simple exchange economy admits a unique efficient allocation. Given that the market protocols attain allocative efficiency, this implies convergence to the same allocation and facilitates comparisons. Following Smith (1982), we identify